Comparison Between Just-in-Time Purchasing and The Economic Order Quantity Model: Mathematical Modeling

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DEDICATION

In special memory of my brother, Marwan, for his shining example that he has left to guide my way. Thank you, Marwan, for the love we shared as brothers, for the integrity that mirrored your life, and the wisdom that you freely shared with me. Thank you for all the beautiful memories that I hold within my heart, and encourage me to live my life each day to its fullest. May I live my life in such a way, that it brings honor to your memory. I love you, my brother. Until we meet again, may God shower you with His blessings.

To my mom and dad, for their unfailing love and support throughout my life, who have guided me to make wise choices during my life and who have inspired me to be all that I can be, without your support I wouldn't have achieved what I have today.

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Nomenclature

D	Annual demand for inventory item (units/year)
H	Annual inventory holding cost per unit ($/unit/year)
Q	Order quantity
K	Ordering cost ($/order)
C_s	Shortage cost per unit ($/unit)
$E(s)$	Expected number of units short per period (units/period)
d_{max}	The maximum demand during the replenishment lead time
Z	Cost difference between EOQ and JIT systems
TAC_E	Total annual cost for the EOQ system
TAC_J	Total annual cost for the JIT system
P_J	Purchase price per unit under JIT system
P_E	Purchase price per unit under EOQ system
OC	Annual ordering cost
HC	Annual holding cost
SC	Annual stock-out cost
C_{ind}	The level of annual demand (D_{ind}) in dollar value
P_{Jmax}	The upper limit purchase price above which JIT will be more costly than EOQ
D_{ind}	Indifference point
D_{max}	Demand level at which JIT's cost advantage is maximized
Q_B	Price break quantity
EOQ	Economic order quantity
OOQ	Optimal order quantity
TBO	Time between orders
π	Quantity discount rate
R	Reorder point in the EOQ model
D	Demand during the replenishment lead time
$P\ (d)$	The probability of a demand during the replenishment lead time
Q_{max}	The minimum quantity needed to get the minimum price level in the linear discount pricing policy
IP	Inventory position

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Abstract

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In recent years, many manufacturing and service industries have been challenged to increase their focus on customer satisfaction and quality of products due to the expansion of global competition. Confronting these challenges of global competition, companies world-wide are forced to find ways to reduce costs, improve quality, and meet the ever changing needs of their customers. One successful solution has been the adoption of just-in-time (JIT) system, which involves many functional areas of a company such as manufacturing, engineering, marketing, and purchasing.

The number of organizations re-examining their production strategies and purchasing policies has been escalating since the early 1980s as a direct result of intensifying global competitiveness pressures. This creates a need for a mathematical model quantifying and then comparing both economic order quantity and JIT systems' related costs. Validating the feasibility for a company to switch to the JIT system arises as a direct result.

In this thesis inventory costs of purchasing under economic order quantity (EOQ) system with fixed, linear discount, and step function discount pricing policies are determined and compared to the just-in-time costs through the development of three mathematical models. The study determined the indifference point (level of demand at which the costs are the same) between the two systems and identified under what conditions one system is superior to the other, from a cost perspective. A user-friendly program was then established to utilize the developed models. The results showed that at low levels of demand, JIT is the preferred method, whereas, EOQ has the cost advantage for an item with a high demand. The models also predicted that the higher the costs associated with the EOQ model (holding, ordering, and stock-out costs), the higher the indifference point will be, and the wider the range of annual demands over which the JIT system is more cost effective.

Chapter One

Introduction

1.1 Introduction

The number of organizations re-examining their production strategies in the manufacturing sector has been steadily escalating since the early 1980s as a direct result of intensifying global competitiveness pressure. In increasingly competitive markets customers have grown more demanding, and this rising customer emphasis on quality further acts as a stimulus for companies to somehow strive for perfection within the organization. One method to which firms have been turning in order to improve productivity, quality, and efficiency is the implementation of the now-familiar Japanese management technology called "just-in-time" (JIT).

The utilization of JIT involves having the correct items of the appropriate quality and quantity in the right place and at the right time. The general principles of JIT are: to produce according to customer demand, with perfect quality, with zero unnecessary lead times; to eliminate waste; to develop the productive potential of a firm's labor force; and to pursue a quest for the continuous improvement of every aspect of the manufacturing operation. The effective implementation of this production technology can lead to greater productivity, lower costs, and a higher profit (Cheng, 1990). These impressive effects of implementing the JIT system, consequently made JIT the target for many companies trying to cope with the increasingly higher quality demands of their customers.

Traditionally, inventory has been viewed as an asset, one that can be converted to cash. In recent years there has been a move towards the use of JIT production methods. The JIT view is that inventory does not add value but instead incurs costs, and thus a waste.

Today, manufacturing companies are adopting an “inventory is waste” philosophy using just-in-time production, which usually combines the elements of total quality control to achieve high productivity. JIT turns the EOQ formula around. Instead of accepting set-up times as fixed, companies work to reduce set-up time and reduce lot-sizes (Jaber and Boney, 1999).

1.2 Problem Definition and Importance

Just-in-time is one of the modern manufacturing systems and its use has assisted many firms in becoming more productive and competitive. JIT is designed to virtually eliminate the need to hold items in inventory. It is defined as: “to produce and deliver finished goods just in time to be sold, sub-assemblies just in time to be assembled into goods, and purchased materials just in time to be transformed into fabricated parts” (Schonberger, 1982).

However, the benefits associated with JIT generally surpass the mere savings in inventory holding costs. A well-implemented JIT system will also result in improved quality, lower manufacturing costs, elimination of waste, and elimination of production process bottlenecks. Most JIT companies view JIT purchasing as a significant component of their JIT implementation and a major factor in their success.

Despite the impressive success of JIT programs, many companies still use the traditional approach to determine their purchase orders. This is particularly true for small manufacturing firms who cannot effectively implement JIT purchasing. The traditional inventory management practices center around the economic order quantity model, which focuses on minimizing the inventory costs, rather than on minimizing the inventory (Johnson and Stice, 1993).

Manufacturing companies that use economic order quantity (EOQ) purchasing, either classical EOQ model or a variation thereof, increasingly are faced with the decision of whether or not to switch to the just-in-time purchasing policy. This is a complex decision, requiring careful examination of each system and its possible impact on a variety of factors, such as cost, quality, and flexibility of the operations. This creates a need for a comparative analysis of these two popular inventory management practices, and an examination of the many factors that enter into such a decision. Based on the above, the main **objective** of this research work is to develop a mathematical model that quantitatively compares the variable costs associated with both the EOQ and JIT models. Thus allowing the assessment of the cost conditions, under which a company would be better of adopting either EOQ or JIT purchasing models.

1.3 Methodology

The research methodology can be described as follows:

1. Reviewing the existing related literature on JIT purchasing and the EOQ model.
2. Analyzing inventory costs affecting both the JIT purchasing and the EOQ model.

3. Analyzing the most popular discount schemes.
4. Developing a mathematical model that quantitatively compares variable inventory costs associated with the EOQ and JIT purchasing models, for the fixed item unit cost, as well as for the presented discount schemes.
5. Assessing cost conditions under which a company would be better of using one of these two models, i.e. either JIT or EOQ model.
6. Developing a user-friendly software to include the developed mathematical models.
7. Discussing the results, coming up with the conclusions and recommendations.

1.4 Organization of The Thesis

This thesis includes six Chapters; the First Chapter provided an overview of the idea behind this research. Theoretical background and literature review will be presented in the Second Chapter. The Third Chapter will include the three developed mathematical models. Chapter Four includes examples along with the discussion of these results. Presenting the developed program with its computer interfaces will be in Chapter Five. Conclusions, and finally suggested recommendations for future researchers will conclude the thesis.

Chapter Two

Theoretical Background and Literature Review

2.1 Introduction

Traditionally, inventory has been viewed as an asset, one that can be converted to cash. In recent years there has been a move towards the use of JIT production methods. JIT view is that inventory does not add value but instead incurs costs, and thus a waste. The JIT concept of continuous improvement applies primarily to a repetitive manufacturing process.

Today, manufacturing companies are adopting an "inventory is waste" philosophy using just-in-time production, which usually combines the elements of total quality control to achieve high productivity. JIT turns the EOQ formula around. Instead of accepting set-up times as fixed, companies work to reduce set-up time and reduce lot-sizes (Jaber and Boney, 1999).

This Chapter begins by introducing the traditional inventory control systems along with their advantages and disadvantages, and then it will proceed through the Economic order quantity (EOQ), and conclude with brief presentation of the two discount schemes that will be taken into consideration in the next Chapter, presenting the mathematical model. Additionally, JIT and much of the literature supporting this concept will be presented, beginning with JIT definition, the main characteristics of the JIT, and the JIT operational benefits will be reviewed.

Since JIT purchasing, represents an important topic with respect to JIT, much of the literature regarding this issue will be cited. JIT in manufacturing, and JIT in service sector also will be presented. The need for a mathematical model to compare between inventory costs of JIT and EOQ system is emphasized in the other Section and will conclude this Chapter.

2.2 *Inventory Control*

The importance of inventory control is well known in any facility, whether it is a manufacturing or a service one. This importance rises from the fact that its level will be set to balance the contravercity of the amount of tied up money in the stores, versus the capability of satisfying any expected demand during any period of time, which is known as the customer satisfaction. Inventory control systems, bring together both dimensions; sizing and timing, since they specify how much and how often a replenishment need to be made.

Inventory can be basically divided into three phases. Starting from the time the facility receives the inventory as raw material, passing through the time the inventory is under operation as work in process, and ending with the time the facility dispatches the inventory to other places as finished goods. There is a great interaction between the three phases of inventory. The finished goods of any facility may be the raw materials or the work in process for another facility and so on. The three phases are:

1. *Raw Materials Inventory:* In which the raw materials of the factory are stored.
2. *Work in Process Inventory:* Which represents the materials stored during adding value to the product (production buffers).
3. *Finished goods inventory:* In which the products are stored after completion of the production process of the product.

2.2.1 *Traditional Inventory Control Systems*

Its good here, in discussing traditional inventory control systems to mention the different types of inventory. According to Krajewski and Ritzman (2002), there are four different types of inventory:

1. *Cycle inventory:*

   It is the portion of total inventory that varies directly with lot size. At the beginning of the interval, the cycle inventory is at its maximum, or equals the order quantity (Q). At the end of the interval, just before a new lot arrives, cycle inventory drops to its minimum, or zero value. The average cycle inventory of these two extremes will be Q/2. This case applies only when the demand rate is constant and uniform.

2. *Safety stock inventory:*

   Its considered to be a safe guard against stocking out during the replenishment lead-time. To create a safety stock, a firm places an order for delivery earlier than when item is typically needed. The replenishment order therefore arrives ahead of time, giving a cushion against uncertainty.

3. *Anticipation Inventory:*

   It is defined as the inventory used to absorb uneven rates of demand or supply, which businesses often face. Predictable seasonal demands patterns lend themselves to the use of such a type of inventory.

4. *Pipeline inventory:*

   Inventory moving from point to point in the materials flow system is called pipeline inventory. Materials move from suppliers to a plant, from one operation to the next in the plant, from the plant distribution center to the customer.

Depending on the type of service provided by the company, or depending on the company's strategy, whenever a stock-out occurs one of two situations may happen:

1. *Backorder:*

   It is a customer order that cannot be filled when promised or demanded but is filled later (Krajewski and Ritzman, 2002). The company may refer to backorders to solve the problem of stock-out, however, this method may reduce customer satisfaction, because of delays in supplying the required quantity.

2. *Loss of sale:*

   In this situation the company will lose the customer if a stock-out occurs. A good example to this kind of companies is the Fast Food Restaurants.

Before discussing the most popular inventory control system, we need to define the inventory position (*IP*). The inventory position measures the item's ability to satisfy future demand. It includes scheduled receipts, which are orders that have been placed but not yet received, plus on-hand inventory minus backorders.

There are several inventory control systems in reality, two of which assume great importance. These systems will be presented in the coming Sections (Krajewski and Ritzman, 2002).

2.2.1.1 Continuous Review System

A continuous review system, sometimes called fixed order quantity system; in which the inventory position (*IP*) is checked whenever there is a transaction, if the (*IP*) reaches or drops below a pre-specified value called the reorder point (*R*), a fixed quantity (*Q*) is ordered. The use of the (*IP*) as the trigger for the replenishment helps to avoid making a replenishment order, if the firm is expecting to receive an order.

For this system to work, it is necessary that *Q* is larger than the demand during the lead-time. This system has the following advantages:

1. It is a simple system, for both implementation and understanding by personnel.
2. There is less likelihood of errors.
3. It is easier for the supplier, because he has a fixed quantity to supply.

Despite the positive advantages, the (*Q*) system has the disadvantage that demand, which triggers the replenishment, may be so large that even when the replenishment quantity *Q* arrives, the *IP* will stay under the reorder point that will cause a stock-out. This situation however can be solved by ordering an integer multiple of *Q*.

2.2.1.2 Periodic Review System

An alternative inventory control system is the periodic review (*P*) system; in which the *IP* is checked every period *P*, and a new order is always placed at the end of each review to make the *IP* reach a pre-specified target inventory level. In this system, the lot size *Q*, may change from one order to the next but the time between orders is fixed. This system has the following advantages:

1. The replenishment of related items (i.e. having the same supplier) may be coordinated.
2. It offers a regular period for the opportunity of modifying the value of the target inventory level.

However, it also has the following disadvantages:

1. The holding costs are usually higher.
2. There is a great uncertainty in whether the *IP* will supply the demand during the period R.

2.2.2 Determining the Replenishment Quantity (Q)

The Economic order quantity is a very important issue, since it determines the order quantity that minimizes the total annual costs. The assumptions under which the relation was derived (Krajewski and Ritzman, 2002):

1. The demand rate for the item is constant.
2. There are no constraints on the size of each lot.
3. The ordering cost is fixed per order.
4. Decisions for one item can be made independently of decisions for other items.
5. The lead-time is constant.
6. The entire order quantity is delivered at the same time.

There are three general components of cost used in computing the total annual costs. They are:

1. Annual inventory ordering cost.

2. Annual inventory holding cost.
3. Annual item purchase cost.

Therefore,

Total annual cost = Annual inventory ordering cost + Annual inventory holding cost + Annual cost of materials (2.1)

The annual holding cost, which increases linearly with Q, can be calculated:

Annual holding cost= (Average cycle inventory) (Unit holding cost) (2.2)

Whereas, the annual ordering cost equal:

Annual ordering cost= (Number of orders per year) (Ordering cost) (2.3)

The average number of orders per year equal annual demand divided by Q.

Therefore, the total annual cost formula will be:

$$TAC = \frac{D}{Q}(K) + \frac{Q}{2}(H) + PD \qquad (2.4)$$

Where:

D: is the annual demand,

Q: lot size in units,

H: cost of holding one unit in inventory for a year.

K: cost of ordering one lot.

P: the item unit purchase price.

And (*PD*) will result in the annual purchase cost.

Differentiating the above equation with respect to the annual demand and then equating the result to zero. Solving for Q will result in the EOQ, and the formula for the EOQ will be:

$$EOQ = \sqrt{\frac{2DK}{H}} \tag{2.5}$$

2.2.3 *Quantity Discounts*

For some service providers and for manufacturers, the cost, per unit for purchased material, sometimes depends on the quantity ordered. The larger the lot size is, the less the unit cost will be. Quantity discount is considered as being an incentive for the buyer to order larger quantities in order to get the benefit of lower unit prices for a larger lot size.

The types of quantity discount considered here is an "all-unit" quantity discount, in which all units ordered in the lot size will have the same unit cost. Two different discount schemes are studied later in our model, the discount models are:

1. *Linear decreasing function*: in which the delivery price function considered in this discount scheme for an inventory item is a decreasing linear function. The unit price decreases linearly with larger order quantities until it reaches a certain quantity. Beyond this quantity unit price will be fixed and at its minimum value.
2. *Step function unit price*, at which the unit price is fixed for a certain range of quantities, then the unit price will change for another certain range of quantities. The price will be fixed for the same quantity range.

2.2.4 *Selecting Time Between Reviews*

Selecting the time between reviews could follow one of the following suggested situations:

- It could be a management's decision. For instance, the manager decides that the time between orders will be each Friday, or every other Friday.
- It could be the average time between orders for the economic order quantity that will be the case if the replenishment quantity is the EOQ, as described before.
- If we use any model other than the EOQ, time between orders can be determined by dividing the lot size by the annual demand and then converting this ratio into months, weeks, or days as needed.

2.3 Just-In-Time

Just-In-Time is a philosophy aimed at minimizing total costs through the pursuit of continuous improvement, the application of specific manufacturing and purchasing techniques, a commitment to total quality and the unlocking of the full potential of an organization's human resources. According to Krajewski and Ritzman (2002), JIT is the organization of resources, information flows, and decision rules that can enable an organization to realize the benefit of the JIT philosophy. They added, JIT is simple but powerful-eliminate waste by cutting unnecessary inventory and removing non-value-added activities in operation. The goals are to produce goods and services as needed and continually improve the value-added benefits of operations.

2.3.1 Characteristics of JIT

The just-in-time system focuses on reducing inefficiency and unproductive time in processes to continuously improve the process and the quality of the products and services they produce. In this Section we discuss the following characteristics of JIT system: pull method of material flow, consistently high quality, small lot sizes, uniform plant loads, standardized components and work methods, close supplier ties, flexible work force, line flows, automated production, and preventive maintenance.

2.3.1.1 Pull Method of Material Flow

A well-known material flow type is the push method, it is the method adopted by traditional manufacturing systems, where the production of the item is produced in advance of customer needs. Whereas, JIT system utilizes the pull method of material flow in which customer demand activates production of the item.

Brox and Fader (1997) argued that the JIT manufacturing system is pull system, which means that the production system responds to customer demand regardless of the level of demand. This is in contrast with the traditional manufacturing push system; where production schedules are determined before demand is precisely know. Ahmadi and Matsuo (2000) stated that pull production lines are characterized by small work-in-process (WIP inventory) and good performance along the metrics of cost, time, and quality.

Firms that tend to have highly repetitive manufacturing processes and well-defined material flows, use JIT systems because the pull method allows closer control of inventory and production at the workstations. Other firms, such as those producing a large variety of products in low volumes with low repeatability in the production process, tend to use the push method. In this case, a customer is promised delivery on some future date. Production is started at the first workstation and pushed ahead to the next one. Inventory can accumulate at each workstation because workstations are responsible for producing many other orders and may be busy at any particular time (Krajewski and Ritzman, 2002).

2.3.1.2 *Consistently High Quality*

Quality is an integral part of a JIT program. Rather than assessing quality control at the final inspection of completed items, the quality control staff is concerned with the prevention rather than the detection of defects. By changing the role of the quality control (QC) staff, many organizations can reduce the number of QC members and assign the traditional responsibilities of QC to Just-in-time manufacturing production departments. The remaining QC staff members work closely with production workers teaching them techniques in statistical quality control (SQC) so they may identify and resolve minor quality problems. Quality control staff also train suppliers in SQC so they can use these techniques to improve their products.

Frazier et al. (1988), extended that the fundamental objective of JIT is to eliminate all waste from the entire supply chain. Producing a quality product in any organization is the goal of JIT systems.

One of the results of JIT systems is the elimination of waste or non value-adding activities; this contributes to quality. The difficult task in achieving quality is achieving the level of quality that the customers require. To be able to give customers the quality they require the company must be committed to a continuous quality improvement program (Sinnamon, 1993). Quality facilitates JIT because poor quality is among the main reasons for maintaining ``just-in-case'' levels of inventory (Dean and Snell, 1991).

2.3.1.3 *Small Lot Sizes*

Haan and Yamamoto (1999) stated that if inventory is waste and product lifetimes decline at a growing speed, a firm should be very cautious about having such inventories. Since JIT practices originated in Japan, if zero inventory management is possible, one should find it in Japanese firms. They added that the main reason to aim for as low an inventory of raw materials and components as possible stems from the costs of such inventories.

Tracey et al. (1995) stated that it has been well documented that the implementation of the just-in-time (JIT) purchasing systems can result, on average, in reduced inventory costs, shorter lead times, and improved productivity for buying organizations

Salameh and Jaber (1997) argued that ever since the economic order quantity inventory control models, known as just-in-case (JIC) models, were introduced in the earliest decades of this century, they have been widely accepted by many industries. New manufacturing concepts, such as JIT production and quality at source have tremendous impact on the productivity and quality in many manufacturing systems. In order to implement the JIT manufacturing concept, one has to analyze its consequences on lot sizing and work in process inventory. The implementation of JIT promises continuous improvement of the manufacturing system. Ideas such as in-process learning, reduction in setups, zero defects, preventive maintenance, etc. are adopted by JIT. Such ideas have induced among researchers the inspiration of putting the EOQ model into context with the JIT philosophy.

2.3.1.4 *Uniform Plant Loads*

The JIT system work best if the daily load on individual workstations is relatively uniform. Uniform loads can be achieved by assembling the same type and number of units each day, thus creating a uniform daily demand at all workstations.

A uniform plant load has two ideas: cycle time and frequency of production. Cycle time deals with the rate of production, and level loading deals with the frequency of production. Cycle time with just-in-time is the measure of the rate of requirements. This is usually measured by the rate of sales. Instead of producing what the machine is capable of producing, a company should preferably produce only to the demand that is needed. The cycle time should be implemented with the last operation in mind; with the last operation being the need of the consumer.

The second part of a uniform plant load is producing the product at the right frequency or level loading. The main idea with level loading is that the product must be produced as frequently as the customer has the need for it. The goal is to produce smaller and smaller amounts with more setups, but without additional setup costs. Finding ways to reduce setup time, and then reinvest the time saved into more frequent setups should do this.

The most important benefits of level loading are that it lays the groundwork for balance by producing the product smoothly and predictably. A company can also learn curve improvements by producing that item every day. The employee gets in the rhythm of the production without production activities changing daily. Another benefit of level loading is increased mix flexibility. If a consumer changes their purchase, a company is more able to successfully fill their order reflecting the order modifications. Lead times will also be reduced with level loading. If a product was produced once a month previously, and now it is produced daily, the consumer will receive the product sooner. By meeting customers' demand in a timely fashion they will have more repeat customers and increase the total number of customers.

The last benefit of level loading is quality improvement. It is a fact that the faster the setup times the better the setup. In addition the more repeatable the process is the more consistent the product will be (Swanson and Lankford, 1998).

2.3.1.5 *Standardized Components and Work Methods*

The standardization of components, also called part commonality or modularity, increases repeatability. The result appears in achieving high productivity and low inventory objectives in JIT systems.

2.3.1.6 *Close Suppliers Ties*

Because JIT systems operate with very low levels of inventory, close relationships with suppliers are necessary. Stock shipments must be frequent, have short lead times, arrive on schedule, and be of high quality. A contract might require a supplier to deliver goods to a factory as often as several times per day. Purchasing managers focus on three areas: reducing the number of suppliers, using local suppliers, and improving supplier relations.

The JIT philosophy mandates a reduction in the number of suppliers. The impact of this reduction is that the quality of the relationship between buyers and suppliers improves as these material "partners" work closely together. Yet another benefit of reducing the number of suppliers is the reduction in paper work. An organization will reap many benefits in terms of costs and time-savings by reducing the number of suppliers and improving relationships with these valued partners in the JIT manufacturing process.

Kelle and Miller (2001) argued that JIT philosophy represents an aggressive approach to dealing with all sources of inefficiency by insisting on a reduction in the number of suppliers so that any sources of uncertainty are exposed and eliminated. On the other hand, dual sourcing represents a defensive response to the uncertainty in procurement lead times. To reap the benefits and yet avoid its disadvantages, many companies are moving towards a smaller supplier base of two or three suppliers. Since the non-interruption of supply is essential to the success of JIT, a policy should be adopted that minimizes the risk of shortage.

The cooperative relationship, on the supplier's side, insulates them from the full force of competition in the supply segment of the market chain. This is particularly noticeable when the supplier is committed to only one, or at most, a few purchasers. The buyer, on the other hand, can benefit from the non-investment and low risks of this "vertical integration". The following contains the main features of the JIT relationship (Swanson and Lankford, 1998):

1. Buyers and suppliers become partners.
2. It is a win-win game for both parties.
3. Primary focus is profit-margin gain for both and equal sharing of the rewards.

   In this win-win environment, communication is an essential ingredient for developing successful purchaser-supplier relations. New or enchanted communication patterns being practiced include, (Richeson *et al.,* 1995):

4. More freely exchanged cost, schedule, and quality control information.
5. The elimination of purchase orders for each shipment.
6. Involving suppliers in the development of design specifications.
7. The formation of joint task forces to resolve concerns.
8. The use of supplier and manufacturer plant visits.
9. The use of electronic data interchange (EDI).

By opening the channels of communication, companies foster the development of trust between the purchaser and its supplier, thereby facilitating openness in the exchange of information.

This transfer of costs may be due to poor implementation of JIT purchasing by suppliers, to poor information flows between buyers and suppliers, or, in general, to poor implementation of JIT purchasing from a supply chain perspective. Therefore, Dong et al. (2001) developed a model to determine whether the use of JIT purchasing reduces costs for both suppliers and buyers. The results indicated that JIT purchasing directly reduces costs only for buyers. An indirect path, however, was found between JIT purchasing and logistic costs for suppliers. To the extent that JIT purchasing may result in suppliers adopting JIT manufacturing techniques, then suppliers too can benefit, at least indirectly, from JIT purchasing. At the end they concluded that in order to implement a successful JIT purchasing program from a supply chain perspective, managers in both the supplier and buyer organizations must act to produce the conditions conducive to JIT adoption and success for both buyers and sellers.

2.3.1.7 *Flexible Workforce*

Swanson and Lankford (1998) mentioned that another important building block in the JIT process is an organization's personnel. Workers are considered assets to an organization utilizing JIT manufacturing and are given more latitude with authority to make decisions.

However, the workers are also expected to perform a more varied role within the company because they are cross-trained to perform several different functions, allowing flexibility in reducing bottlenecks as well as substituting for absent co-workers. The workers in a JIT facility are considered experts in the processes they perform and hence become an important part of the JIT team.

In more recent times there has been an increase in research into the effect on and importance of the human variable to the success of JIT. Power and Sohal (2000) examined current human resource management practice in three Australian companies that have been operating the JIT methodology for some years. The cases focus on practices in the areas of levels of participation, multi-skilling and flexibility, communication, employee development programs, teams, and empowerment. The conclusions reached from the research include: (1) communication in JIT companies can be expected to be open, direct and less formal; (2) JIT companies can be expected to be characterized by a participative management style; (3) a strategy of empowering employees is central to the effective operation of the JIT methodology; (4) JIT companies need to actively promote the development of a multi-skilled and flexible workforce; (5) team based structures are common in JIT environments and can be expected to be used as a driver for continuous improvement processes; (6) effective employee development programs are important in JIT environments to underpin other strategic elements; (6) there is evidence to suggest that the combination and emphasis of the overall human resource strategy employed in the JIT environment is potentially more important than the individual elements; (7) JIT environments can be characterized as dynamic systems requiring awareness and management of change processes. The research indicated that the ability to tap into and maximize the human potential of the organization would be a major determinant of the success, or otherwise, of continuous improvement processes, and therefore the just-in-time methodology.

2.3.1.8 Line Flows

Line flows can reduce the frequency of setups. If volumes of specific products are large enough, groups of machines and workers can be organized into a product layout to eliminate setups entirely. If volume is insufficient to keep a line of similar products busy, group technology can be used to design small production lines that manufacture, in volume, families of components with common attributes. Changeovers from a component in one product family to the next component in the same family are minimal (Krajewski and Ritzman, 2002).

2.3.1.9 Automated Production

Automation plays a big role in JIT systems and is a key to low-cost operations. Money freed up because of JIT inventory reductions or other efficiencies can be invested in automation to reduce costs. The benefits, of course, are greater profits, greater market share (because prices can be cut), or both. Automation should be planned carefully.

Chandra and Kodali (1998) argued that automation is an important issue of JIT manufacturing as it applies to the organization's capacity to manufacture in an efficient manner. It involves the changing or adaptation of machinery and processes to a company's specific manufacturing needs. The greater the degree an organization is able to automate, the greater the competitive edge it will offer.

2.3.1.10 *Preventive maintenance*

An organization without preventive maintenance operates heavily under the risk of facing accidents, safety problems, substantial repair costs and out-of-control manufacturing processes. Preventive maintenance is not solely the responsibility of one individual department. Effective maintenance policies such as preventive maintenance, total productive maintenance, etc., should be implemented in companies that utilize JIT.

Because JIT emphasizes finely tuned flows of materials and little buffer inventory between workstations, unplanned machine downtime can be disruptive. Preventive maintenance can reduce the frequency and duration of machine downtime. After performing routine maintenance activities, the technician can test other parts that might need be replaced. Replacement during regularly scheduled maintenance periods is easier and quicker than dealing with machine failures during production. Maintenance is done on a schedule that balances the cost of the preventive maintenance program against the risks and costs of machine failure (Krajewski and Ritzman, 2002).

2.3.2 *JIT Operational Benefits*

Just-in-time systems have many operational benefits. They include (Krajewski and Ritzman, 2002):

1. Reduce space requirements
2. Reduce inventory investment in purchased parts, raw materials; work in process, and finished goods.
3. Reduce lead times.
4. Increase the productivity of direct-labor employees, indirect-support employees, and clerical staff.
5. Increase equipment utilization.
6. Reduce paperwork and require only simple planning systems.
7. Set valid priorities for scheduling.
8. Encourage participation by the workforce.
9. Increase product or service quality.

2.3.3 *JIT Purchasing*

JIT purchasing means providing materials to the production facility just as they are required for use. It goes against most traditional ideas held by manufacturing, purchasing, and material management. JIT purchasing is integral and is typically incorporated when describing JIT management practices.

The precision involving quality, timing and quantity required of operations within a JIT manufacturing system is equally important for operation upstream from the JIT manufacturer. With JIT purchasing, suppliers become extended operations of the JIT manufacturer. The importance of JIT purchasing in the overall JIT management system is demonstrated by the magnitude of parts by a typical manufacturer.

The fundamental aim of JIT purchasing is to ensure that production is as close as possible to a continuous process from receipt of raw materials/ components to the shipment of finished goods. The success and resulting performance of purchasing system is based upon cooperation between the purchaser and supplier. Some of the elements of this system are as follows (Gunasekaran, 1999):

1. smoothed flow of materials between suppliers and buyers,
2. order cost reduction,
3. stock reduction,
4. quality, and
5. product simplification.

A number of articles have been published on the practice of JIT purchasing, however, most are descriptive in nature but leave little in the way of specific and implementable ideas. They are simply dealing with one or two methods installed in an environment about which the reader is given in-depth research leading to a more insightful understanding of the JIT purchasing concept.

Gunasekaran (1999) stated that the JIT purchasing concept attempts to reduce replenishment lead-time by utilizing suppliers located close to the using plant and by ordering small quantities, which in turn reduces a supplier's workload per period. He added that the most important aspects of the JIT purchasing concept focus on new ways of dealing with suppliers and a clear-cut recognition of the appropriate purchasing role in the development of corporate strategy. Suppliers should be viewed as 'outside partners' who can contribute to the long-run welfare of the buying firm rather than as outside adversaries. The major actions focus on attempts to reduce the ordering cost and replenishment lead-time values.

A conceptual model is presented in Figure (2-1) to illustrate the JIT purchasing system and the major issues involved with reference to suppliers and buyers linkages (Gunasekaran, 1999).

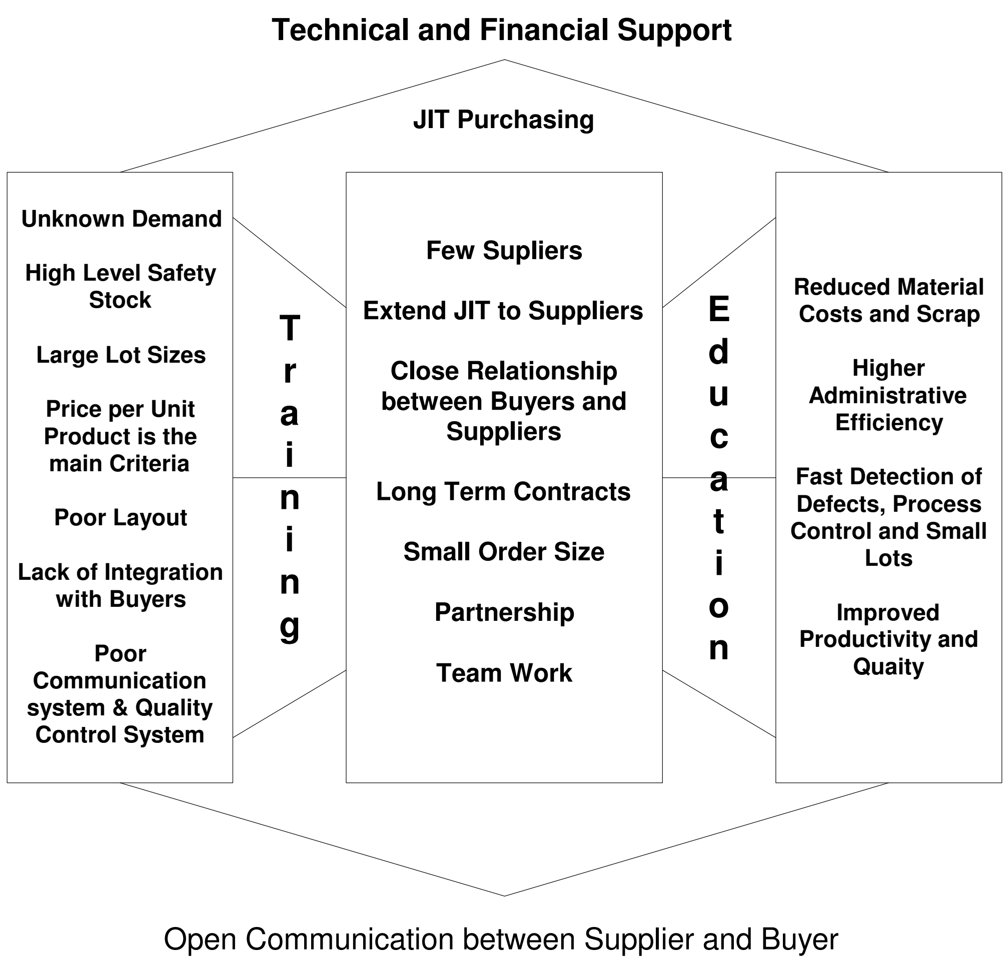

Figure (2-1) A Conceptual model to illustrate the supplier-buyer relationship under JIT purchasing.

Figure (2-1) shows that the characteristics of JIT purchasing include: few suppliers, nearby suppliers, frequent deliveries in small lot quantities, long-term contract agreements, close relationships between buyers and suppliers, and the use of company-owned or contract shipping. The buyers benefits of JIT purchasing are: reduced cost for parts, few suppliers to contract with, reduced expediting, reduced travel and telephone costs, fast detection of defects, less need for inspection (of lots), quick response to engineering changes, and reduced rework, late deliveries, production control and supervision.

2.3.4 JIT in Manufacturing

Owing to its relatively small geographical area, Japan was forced to find ways to efficiently use its scarce resources. The Japanese have turned these disadvantages into advantages by successfully developing and implementing JIT production systems.

They view the manufacturing process as a network of linked work centers where the optimal arrangement enables each worker to finish his or her task and deliver it to the next worker exactly when it is needed. The ultimate goal is to completely eliminate all waiting time so that inventory investment can be minimized, production lead times can be shortened, demand changes can be quickly addressed, and quality problems can be uncovered, and solved.

JIT can be seen as a new way of thinking, planning, and performing with respect to manufacturing. The basic principle of JIT in manufacturing is to eliminate all forms of waste, defined as anything that does not add value to the product (Burnham, 1987). The first step is to identify activities that are considered to be waste-producing. The major areas for different forms of waste that may be present in many departments according to Hernandez (1989) are:

1. Waste in the production line.
2. Waste in the materials department.
3. Waste involving suppliers.
4. Waste in design engineering.
5. Waste from waiting.
6. Waste from transportation.
7. Waste from defective parts.

In summary, the objective of JIT can be simply stated as ``produce the right item, at the right time, in the right quantities''. By achieving this objective, companies work toward the elimination of waste in their manufacturing processes and realize the following benefits (Chase et al. 1998):

1. Lower raw material, work-in-process, and finished goods inventories.
2. Higher levels of product quality.
3. Increased flexibility and ability to meet customer demands.
4. Lower overall manufacturing costs.
5. Increased employee involvement.

2.3.5 JIT in Service Sector

Let us start by defining the term "service operation". The definitions and descriptions found in the literature for this are somewhat ambiguous. It is typically easier to describe service operations by what they are not. For example, Lovelock (1984) defines services as ``all those economic activities in which the primary output is neither a product nor a construction''. This definition seems straightforward, but is not particularly helpful when one attempts to classify a restaurant, or a company such as IBM, for that matter.

IBM manufactures equipment but also provides customer service, education, maintenance, etc.

We think about service in humanistic terms; we think about manufacturing in technocratic terms. This is why manufacturing industries are considered to be progressive and efficient while service industries are, by comparison, primitive and inefficient (Canel et al. 2000)

For services, inputs are the customers themselves. Customers typically arrive at their own discretion, with unique demands on the service system. Resources (i.e. goods, labor, capital) are applied by the service manager to facilitate interaction with the customer.

Many of the JIT techniques used by manufacturing firms can be successfully applied by service organizations (Chase et al. 1998). As in manufacturing, the suitability of each technique to the corresponding work process depends on the characteristics of the company's markets, production technology, skill levels, and the corporate culture.

Services are much like manufacturing, in that both employ processes that add value to the basic inputs used to create the final product. JIT focuses on the process, not the product. It can therefore, be applied to any group of processes, whether manufacturing or service. The philosophy behind JIT is to continuously seek ways to make processes more efficient. The ultimate goal of JIT is to produce a good or a service without waste. This goal is approached by testing each step in a process to determine if it adds value to the product or to the service. If the step does not add value, then, it is examined closely to determine possible alternatives. In this way, each process gradually and continually improves. Thus, one of the key requirements of JIT is the constant and continual testing of processes, whether they are in manufacturing or in services (Canel et al. 2000).

2.4 The Need for Mathematical Models

There is a few number of studies comparing EOQ and JIT systems. Most of them advocate the use of JIT over EOQ system. However, in a comparison of JIT and EOQ; Johnson and Stice (1993) concluded that: "traditional inventory management techniques may under-emphasize the costs of maintaining large inventories. JIT may under-emphasize the costs of not maintaining inventories, particularly since such costs are often difficult to identify and measure".

Fazel (1997) conducted a comparative analysis that compared the classical EOQ system with JIT purchasing and determined the cost indifference point between the two systems. In his analysis inventory holding, ordering, and purchased materials costs were the costs constituting the EOQ system annual costs, he did not include the stock-out cost in his analysis. This model took into consideration the case where there is no quantity discount, i.e. the price is fixed. He concluded that increasing the costs associated with the EOQ system would increase cost indifference point.

In another study, Fazel et al. (1998) determined inventory costs of purchasing under the EOQ system with a linear quantity discount and compared these costs to the costs under JIT. They did not take into account the stock-out cost when they determined total annual costs under EOQ system. They only studied the case when the optimal order quantity is the one that belongs to the discount range of order quantities. At the end they concluded that increasing the value of the item, the ordering cost, holding cost associated with the EOQ system will make JIT system preferred over EOQ system.

Based on the above literature a need for mathematical models that evaluate and quantitavely compare different costs associated with the EOQ and JIT models arises, where, EOQ system stock-out cost is quantified and thus added to the models. Three different models will be developed in this research taking into consideration three cases. The first model will focus upon the supplier that offers the JIT buyer a fixed price. The second model will look at the offer of a quantity discount linearly proportional to the order quantity, and finally the third model will review the offer of a step function quantity discount by the supplier.

Chapter Three

Comparison Between JIT Purchasing and EOQ Purchasing

(Mathematical Models)

3.1 Introduction

Just-in-time has assisted many companies in becoming more productive and competitive. The benefits associated with JIT include savings in inventory costs, savings in manufacturing costs, reduction in ordering costs, improved quality, elimination of waste, and the elimination of production process bottlenecks. Despite these impressive advantages of JIT, many companies still use the economic order quantity model to determine their purchase order. The reason behind it is that many companies still cannot effectively use JIT purchasing.

According to EOQ model, a manufacturer places several orders to its supplier every year, with a lot size equals to the EOQ, which is the quantity that minimizes the total costs for the buyer. This most economic quantity can be obtained mathematically through a certain formula, but it differs according to the total annual cost function.

Manufacturing companies that still use economic order quantity purchasing, either the classical model or a variation, increasingly are faced with the decision whether to switch to Just-in-time system, or not. Therefore, such a decision should be based on a careful examination of the many related factors affecting both systems.

This creates a need for a comparative analysis of these two inventory management systems, and an examination of related factors that enter into such a decision. Quantifying and comparing the costs related for both systems will be the core for developing the desired mathematical model.

In this Chapter a mathematical model to compare the total purchasing and inventory costs associated with JIT and EOQ is developed. A fixed cost model (the inventory item cost is fixed no matter how large is order quantity) is developed to compare the total costs associated with both systems. The model will determine, for every item, the demand level at which both of the item's total costs are the same. The model also determines the level of demand where the cost advantage of the JIT system is maximized. Finally the model can also identify under what conditions it would be advantageous for a company to use one of these two inventory management systems, and whether it is feasible to switch to JIT system or not.

Model II and model III expand the classical economic order quantity model to include an all unit quantity discount. Model II studies the case when the delivery price function for an inventory item is a decreasing linear function, where the slope of the decreasing line is considered as being the discount rate. Three different cases appear likely to happen in this model; they are reviewed and included in the model, in addition all the different analysis taken in model I are carried out in model II.

In model III a step function discount scheme is studied. A computational procedure to determine the optimal order quantity is developed, taking this optimal quantity; an analytical model is developed to determine which system (EOQ or JIT) will be less costly.

3.2 Model I

This model takes into account the case where there is no quantity discount for the purchase price under EOQ model, i.e. the delivery price is assumed to be fixed regardless of the quantity ordered by the customer. Therefore, no matter how large the order quantity is, the purchase price will be fixed.

Taking into account that the item purchase price is fixed, a mathematical model will be developed here in the foregoing Sections, comparing the total annual costs for both JIT and EOQ systems, and the break-even demand is found at which the total annual costs for both systems at this value of demand will be the same. Then the highest price at which the buyer under JIT can buy at, and still less costly than using the EOQ system, will be determined. The demand level at which JIT cost advantage will be maximized over the EOQ will also be determined.

3.2.1 EOQ Assumptions

The materials purchased by a company may have a regular consumption pattern or be consumed irregularly. It is the regularly consumed items that account for most of the purchasing and inventory costs. For these items annual demand and consumption patterns can be determined in advance and used as the basis for negotiation with suppliers. The economic order quantity model is most suited for determining the order size for such items.

The approach to determining the EOQ in this model is based on several assumptions. These assumptions include: The annual demand for the item is known and constant, ordering cost is fixed per order, holding cost for the inventory item is constant on a per unit basis, there are no constraints on the size of each lot ordered, and finally the inventory capacity is unlimited. It is also assumed that the only relevant costs are: annual inventory ordering cost, annual inventory holding cost, and the annual delivered goods cost.

3.2.2 Price Function for EOQ

In practice many companies give a fixed delivery price, no matter how large the quantity ordered is. This EOQ price function can be shown graphically in Figure (3-1), where the item unit price is P_E.

Figure (3-1) EOQ fixed price function

3.2.3 EOQ Costs

In the EOQ model the total annual cost of an item *(TAC_E)* is the sum of the annual costs which are: annual inventory ordering cost, annual inventory holding cost, annual stock-out cost and the annual delivered goods cost.

Total annual cost = Annual inventory ordering cost + Annual inventory Holding cost +Annual stock-out cost + Annual cost of materials (3.1)

1. *Ordering costs:* is the cost of preparing a purchase order for a supplier or a production order for the shop. For the same item the ordering cost

is the same, regardless of the order size. The ordering cost can be determined as follows:

Annual ordering cost = (Number of orders/year)(Ordering cost)

$$OC = \frac{D}{Q}(K) \qquad (3.2)$$

2. *Holding cost:* is the variable cost of keeping items on hand, including interest, storage and handling, taxes, insurance, and shrinkage. Holding cost can be determined by:

Annual inventory holding cost = (Average cycle inventory)(Unit annual holding cost)

$$HC = \frac{Q}{2}(H) \qquad (3.3)$$

3. *Stock-out cost:* a stock-out is the situation that occurs when an item which is typically stocked is not available to satisfy a demand the moment the demand occurs, resulting in loss of the sale, and therefore, it causes the customer to go elsewhere. According to Vollmann et al. (1997), stock-out cost can be determined through the following formula:

Annual stock-out cost = (Number of orders/year)(Shortage cost/unit) (Expected numbers of units' short/period)

$$SC = \frac{D}{Q} \cdot C_s \cdot E(s) \qquad (3.4)$$

And,

$$E(s) = \sum_{d=R+1}^{d\,max} (d - R)P(d) \qquad (3.5)$$

Where:

D: annual demand for inventory item (units/year).

Q: order quantity.

H: annual inventory holding cost per unit ($/unit/year).

K: ordering cost ($/order).

C_s: shortage cost per unit ($/unit).

E(s): expected number of units short per period (units/period).

R: reorder point in the EOQ model.

d: demand during the replenishment lead time.

P(d): the probability of a demand during the replenishment lead time.

d_{max}: the maximum demand during the replenishment lead time.

In the models developed, we assume that the shortage cost per unit is fixed and the expected number of units short per order is also fixed and known.

4. *Cost of purchased materials:* simply the cost of purchased materials can be determined by multiplying the item's delivery unit price (P_E) the annual demand of that item.

Summing up all the above costs, will result in the total annual cost for the EOQ system (TAC_E), yielding:

$$TAC_E = \frac{D}{Q}(K) + \frac{Q}{2}(H) + \frac{D}{Q}[C_s E(s)] + P_E D \tag{3.6}$$

3.2.4 Determining the EOQ

The EOQ model has been a popular method for estimating the most economic order size that would minimize the total cost. The optimal order quantity (Q^*) is given by (Vollmann et al. 1997):

$$Q^* = \sqrt{\frac{2D[K + C_s E(s)]}{H}} \tag{3.7}$$

This formula is obtained through calculus: we take the first derivative of the total annual cost function, from equation (3.6), with respect to *Q*, and then equating the derivative with zero. Solving for *Q* will yield the above formula for the optimal order quantity. However, the total annual cost that determines the classical EOQ formula does not include the stock-out cost, but in the foregoing context the quantity resulting from the derivation of the total annual cost that includes the stock-out cost will also be called the EOQ.

Substituting Q* from equation (3.7) in the Total Annual Cost equation (3.6) will result in:

$$TAC_E = \sqrt{2DH[K + C_S E(s)]} + P_E D \qquad (3.8)$$

3.2.5 JIT Assumptions

Ideally, under just-in-time model some of the traditional costs associated with the EOQ model are either eliminated or substantially reduced. These costs include ordering, storage, cost of capital, insurance, and transportation costs. The supplier to a JIT buyer is strongly encouraged to implement JIT in his/her production facility to further reduce costs, improve quality, and become more responsive to the buyer. Therefore, the manufacturer will be economically better off to choose JIT over EOQ since JIT may result in a reduction in purchase price, holding costs, and ordering costs.

Much literature in the field of JIT indicate that for many companies the reality is different from the ideal situation. In practice, many suppliers of JIT manufacturers produce their products in large batches and respond to the JIT challenge by keeping large quantities of items in their inventories even though they may deliver them in small quantities.

Quality control, inspection, and transportation arrangements will also become the responsibility of the JIT supplier, which if not properly managed could add to the supplier's costs. Thus, in these cases, much of the inventory costs of the manufacturer is practically transferred to the supplier.

Therefore, it is reasonable to assume that, in the absence of a holistic system in which both suppliers and manufacturer operate under a JIT system, the supplier would pass some of the costs to the JIT manufacturer in the form of higher item purchase price. The purchase price reflects the item cost as well as, at least partially, inventory ordering and holding costs. This translates into a higher per unit purchase (delivery) price for the JIT manufacturer.

In this model it is assumed that much of the holding and ordering costs are passed on to the manufacturer. Therefore, the buyer has to pay somewhat a higher price to buy an item on a JIT basis, compared to purchases made based on EOQ.

It is also assumed in this model that under JIT, the price per unit over the course of a year may be negotiated and might change, but it is assumed in this model that the price per item will stay the same for each delivery. The delivery price, therefore, is assumed to be a constant (P_J).

3.2.6 JIT Costs

Under the JIT system, much of the holding costs and ordering costs (e.g. preparation of purchase orders for each delivery) can be significantly reduced or eliminated. Other costs such as transportation and inspection costs can be reduced by having the suppliers locate near the buyer's plant or by improving the quality at the suppliers' facilities. The remaining costs associated with holding or ordering items (e.g. storage, inspection, or transportation costs that have not been eliminated) are transferred to the suppliers and are in turn charged indirectly to the buyer as a part of the purchase price.

Therefore, the annual cost to the buyer under JIT purchasing is the product of the annual demand (*D*) and the unit price (P_J), which is assumed to be fixed, where P_J includes the portion of holding and ordering costs that are passed on to the buyer. Therefore,

$$TAC_J = P_J D \tag{3.9}$$

The same JIT assumptions and costs used in this model (Model I) will be used in model II and model III as well.

3.2.7 Cost Difference

Let *Z* represent the difference between the total annual costs of the EOQ and JIT systems, then:

$$Z = TAC_E - TAC_J \tag{3.10}$$

Substituting equation (3.9) and equation (3.8) for the total annual costs of JIT model and the EOQ model, respectively, in equation (3.10) resulting:

$$Z = \sqrt{2DH[K + C_S E(s)]} - D(P_J - P_E) \tag{3.11}$$

Let C be the dollar value of the annual demand, then

$$C = D P_E \tag{3.12}$$

Rearranging equation (3.11) by multiplying and dividing the equation by P_E leads to:

$$Z = \sqrt{\frac{2H[K + C_S E(s)]}{P_E} C} - \left(\frac{P_J}{P_E} - 1\right) C \tag{3.13}$$

3.2.8 The Indifference Point

EOQ model will be preferred over JIT model if Z in equation (3.13) is less that zero. Whereas JIT will be less costly if $Z>0$.

The demand resulting from making Z=0 in equation (3.13), where the total annual cost for JIT and EOQ models are equal, is the indifference point (D_{ind}). This can be achieved by setting $Z=0$ in equation (3.13), where $TAC_E=TAC_J$, solving for C where the resulting $C= C_{ind}$ yields the following:

$$C_{ind} = \frac{\dfrac{2H[K + C_S E(s)]}{P_E}}{\left(\dfrac{P_J}{P_E} - 1\right)^2} \qquad (3.14)$$

Where, the indifference point C_{ind} is the level of annual demand in dollar value, at which the total cost of EOQ and JIT are equal.

Recall C_{ind} from equation (3.12), where $C_{ind} = D_{ind}\, P_E$, and D_{ind} is the annual demand when $Z=0$, then:

$$D_{ind} = \frac{2H[K + C_S E(s)]}{(P_J - P_E)^2} \qquad (3.15)$$

Note that if the annual demand $D > D_{ind}$ then $Z < 0$, so EOQ model will be more cost effective than JIT model, since the total annual cost of the EOQ will be less than the total annual cost of JIT system. Conversely, if $D < D_{ind}$, obviously will be $Z > 0$, consequently, JIT system will be preferred over the EOQ system.

Also note that D_{ind}, the break-even demand, is directly proportional to the holding, ordering, and shortage costs. Making JIT a better alternative for a wider range of demand for a certain item.

3.2.9 Maximum JIT Purchase Price

Maximum JIT price (P_{Jmax}) is the highest price at which the buyer can use JIT model and still be less costly than using the EOQ model, beyond this price EOQ model will be preferred over JIT.

P_{Jmax} can be obtained by setting $Z=0$ in equation (3.11). Then solving for P_J where $P_J=P_{Jmax}$, therefore,

$$P_{J\,max} = \sqrt{\frac{2H[K + C_S E(s)]}{D}} + P_E \qquad (3.16)$$

Obviously, for a given demand D, P_{Jmax} is the highest price that a manufacturer can pay to purchase the item on a JIT basis and still be economically better off than using EOQ purchasing. Whereas, prices higher than P_{Jmax}, Z will be negative, making EOQ a lower cost alternative, and therefore, will be preferred over JIT.

3.2.10 Maximum Cost Advantage

The cost difference between EOQ and JIT system is maximized for a demand level (D_{max}) at which, $dZ / dD=0$.

Therefore, differentiating equation (3.13), and then equating the result with zero and solving for the annual demand where $D= D_{max}$, yields the following:

$$D_{max}=D_{ind} / 4 \qquad (3.17)$$

3.3 Model II

This model deals with the variable of a price discount for items purchased under the EOQ system, unlike the first model where it is assumed that the price is fixed regardless of how large the ordered quantity is.

The delivery price function taken in this model is a linear function, decreasing with a slope equals to the discount rate ($-\pi_E$). The unit price goes down with larger order quantities until it reaches a certain quantity called Q_{max}, beyond this quantity the price is fixed and with a minimum delivery price no matter how large the ordered quantity is, as long as it its larger than Q_{max}.

3.3.1 EOQ Assumptions

The assumptions utilized here in this model are the same basic assumptions used in model I. Whereas, the assumption underlying the EOQ system, that the unit cost remains fixed over the range of order quantities considered, will be relaxed here in this model and Model III.

3.3.2 Price Functions for EOQ

In real life, many companies offer a discount for larger quantities ordered, the larger the quantity ordered, the lower the price will be. The idea behind this discount is to encourage the customer to buy in large amounts, since the costs incurred by the suppliers is usually decreased with the larger lot sizes.

The delivery price function considered in this model for an inventory item is a decreasing linear function. The properties of this quantity discount function is graphically presented in Figure (3-2):

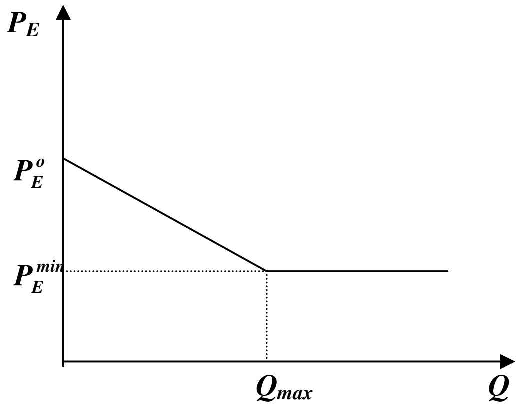

Figure (3-2) EOQ linear price-discount function

Apparently, for quantities less than (Q_{max}) the delivery price will be decreasing continuously and linearly according to the order quantity. Whereas, quantities beyond (Q_{max}), the price is fixed ($P_E^{\min}$) and at its minimum value, which is the lowest price the supplier can offer to the customer regardless how large the order quantity is, and as long as it equals or exceeds Q_{max}. Its good to point out that this type of quantity discount is an all unit discount, i.e. the buyer pays the same price for all the units purchased.

The slope of the discount function in Figure (3-2) represents the quantity discount rate ($-\pi_E$), i.e. the rate at which the price of the item decreases with the increase of the order quantity, and it can be presented as follows:

$$\frac{dP_E}{dQ} = -\pi_E \qquad for\ Q \leq Q_{max} \qquad (3.18)$$

Mathematically the price discount function can be presented as follows:

$$P_E = P_E^{min} \qquad for\ Q \geq Q_{max} \qquad (3.19)$$

$$P_E = P_E^o - \pi_E Q \qquad for\ \ Q \leq Q_{max} \qquad (3.20)$$

Where P_E is the delivery unit price, Q is the order quantity of the inventory item, and $P_E^{\min}$ is the delivery price of the item if the order quantity exceeds Q_{max} and it is fixed.

3.3.3 EOQ Costs

The costs taken into consideration in this model using EOQ basis are: inventory holding cost, inventory ordering cost, stock-out cost, and the delivered goods cost.

In this model we have two situations, one in which the order quantity is less than Q_{max} where the delivery price function is presented in equation (3.20). The other scenario is when the order quantity is larger than Q_{max}, where the delivery price is fixed and equals to $P_E^{\min}$.

Substitute the delivery price function of the first case for the cost of purchased materials in the total annual cost formula in equation (3.6). Therefore the total annual cost for the first case will result in:

$$TAC_E = \frac{D}{Q}(K) + \frac{Q}{2}(H) + \frac{D}{Q}[C_s E(s)] + \left(P_E^o - \pi_E Q\right)D \quad for\ Q \le Q_{max} \qquad (3.21)$$

The same goes for the second case. Therefore,

$$TAC_E = \frac{D}{Q}(K) + \frac{Q}{2}(H) + \frac{D}{Q}[C_s E(s)] + \left(P_E^{min}\right)D \qquad for\ Q \ge Q_{max} \qquad (3.22)$$

3.3.4 Determining The EOQ

The optimal order quantity is the quantity that minimizes the sum of the total annual costs. Since we have two different cost functions for the price function considered in this model, we might find two different economic order quantities Q^* and Q^{**}.

*Q** is the EOQ corresponding to the region where $Q \le Q_{max}$, where *Q** can be obtained by differentiating equation (3.21) resulting in:

$$Q^* = \sqrt{\frac{2D[K + C_s E(s)]}{H - 2\pi_E D}} \qquad for\ Q \le Q_{max} \qquad (3.23)$$

Note that *Q** is real only when $(H - 2\pi_E D) \ge 0$, and *Q** is considered to be an optimum quantity (feasible solution) only if $Q^* \le Q_{max}$.

The other case is when $Q \ge Q_{max}$, the same procedures will be taken in determining *Q** so as to obtain the formula of the EOQ for this range of quantity. Therefore (*Q***) will be:

$$Q^{**} = \sqrt{\frac{2D[K + C_s E(s)]}{H}} \qquad for\ Q \ge Q_{max} \qquad (3.24)$$

Note that the formula of *Q*** is the same formula used in the first model where the delivery price is fixed, and in this case the price is fixed and equals to P_E^{min}. *Q*** is considered to be a feasible solution only if $Q^{**} \ge Q_{max}$, i.e. *Q*** falls within the correct range of *Q*, where the purchase price is P_E^{min}.

Since there should be only one optimal order quantity, and because we might obtain from calculus two different economic order quantities, each belongs to a certain range of quantities, we need to establish a computational procedure with several steps to obtain this one optimum order quantity.

The following steps can be used to find the optimum order quantity:

1. Calculate the economic order quantity using minimum unit cost P_E^{min}, i.e. calculate *Q***.
2. If this quantity is feasible, i.e. the quantity $Q^{**} \ge Q_{max}$, it's a valid economic order quantity and will result in the minimum cost for a

particular item according to EOQ model and will be taken as the optimal order quantity.

3. If Q^{**} is not a feasible quantity, calculate Q^*.
4. Calculate the total annual cost for both Q^* and Q_{max} from equation (3.21).
5. The minimum-cost order quantity is the one associated with the lowest cost in step 4, and will be considered the optimal order quantity.

After determining the optimal order quantity, we need to know which system is less costly than the other, the EOQ or JIT. Therefore, we need to compare the costs for both systems and find the demand level at which both systems' costs are equal. Comparing this demand level with the company's annual demand will assist the manager in making the decision, whether he should switch to JIT system or not.

In this model we will be dealing with three different cases. The first case is when the optimal order quantity is Q^{**}, whereas the second is when the optimal order quantity is Q^*. Finally when Q_{max} is the optimal order quantity will be the third case.

3.3.5 Case I

Case I is where Q^{**} is the optimal order quantity, i.e. Q^{**} is a feasible quantity. In this case note that the delivery price is fixed and equals to $P_E^{\min}$. This case is consistent with *model I*, therefore, we can apply model I for this case using $P_E^{\min}$ as the delivery price. Note that the equation of the EOQ for both *model I* equation (3.7) is the same as the equation used for obtaining Q^{**} in equation (3.24) for this model.

3.3.6 Case II

In case II, Q^{**} is not a valid quantity, whereas, Q^* is a feasible one, and the total annual cost for Q^* is less than the total annual cost of Q_{max}.

The same JIT assumptions and costs used in *model I* can be used for this case, therefore the total annual cost under JIT will be:

$$TAC_J = P_J D \tag{3.25}$$

3.3.6.1 Cost Difference

The total annual cost that will be used in this case under EOQ is presented in equation (3.21), for $Q \leq Q_{max}$. Now, let Z represent the difference between the costs of EOQ and JIT, then:

$$Z = TAC_E - TAC_J \tag{3.26}$$

Substituting equations (3.21) and (3.25) into the above equation results in:

$$Z = \frac{D}{Q}(K) + \frac{Q}{2}(H) + \frac{D}{Q}[C_s E(s)] + \left(P_E^o - \pi_E Q\right)D - P_J D \tag{3.27}$$

Substituting the formula of the EOQ in equation (3.24) for Q in the above equation, the cost different function will be:

$$Z = D\left(K + C_S E(s)\right)\sqrt{\frac{H - 2\pi_E D}{2D[K + C_S E(s)]}} + \frac{H}{2}\sqrt{\frac{2D[K + C_S E(s)]}{H - 2\pi_E D}} + \left(P_E^o - \pi_E \sqrt{\frac{2D[K + C_S E(s)]}{H - 2\pi_E D}}\right)D - P_J D \tag{3.28}$$

Note that Z is real only if $(H - 2\pi_E D) > 0$.

3.3.6.2 The Indifference Point

It is expected that there exists a demand level at which the costs are the same under JIT and EOQ. This demand level can be obtained by equating the cost difference (Z) in equation (3.28) with zero, and the resulting demand is referred to as the indifference point.

Making *Z=0* in equation (3.28) and solving for *D*, yielding:

$$D_{ind} = \frac{2H\left(K + C_S E(s)\right)}{\left(P_J - P_E^o\right)^2 + 4\pi_E\left(K + C_S E(s)\right)} \qquad (3.29)$$

Obviously, from equation (3.29), the break-even demand is directly proportional to the holding cost, ordering cost, and shortage cost. This confirms the expectation that when the inventory cost and the ordering costs are high, JIT system will be more attractive.

3.3.6.3 *Maximum Cost Advantage*

Its expected that there exists a demand level (D_{max}) at which the cost difference between EOQ and JIT is maximized. This demand level can be obtained mathematically by differentiating the cost difference function with respect to the demand, i.e. by obtaining (*dZ / dD*), and then equating the result with zero.

Solving *dZ / dD =o,* for D_{max}, yields the following:

$$D_{max} = \frac{H}{4\pi_E}\left[1 - \frac{P_J - P_E^o}{\sqrt{\left(P_J - P_E^o\right)^2 + 4\pi_E\left(K + C_S E(s)\right)}}\right] \qquad (3.30)$$

3.3.7 *Case III*

In case III, *Q*** is not a feasible quantity, and the total annual cost of Q_{max} is less that the total annual cost of *Q**. Therefore the optimal order quantity that will be used here in this case is Q_{max}.

3.3.7.1 *Cost Difference*

The total annual cost under EOQ is presented in equation (3.22), where $Q = Q_{max}$,therefore,

$$TAC_E = \frac{D}{Q_{max}}(K) + \frac{Q_{max}}{2}(H) + \frac{D}{Q_{max}}[C_s E(s)] + \left(P_E^{min}\right)D \quad for\ Q = Q_{max} \quad (3.31)$$

Let Z represent the difference between the costs of EOQ and JIT, then:

$$Z = TAC_E - TAC_J \quad (3.32)$$

Substituting equations (3.31) and (3.25) into the above equation results in:

$$Z = \frac{D}{Q_{max}}(K) + \frac{Q_{max}}{2}(H) + \frac{D}{Q_{max}}[C_s E(s)] + \left(P_E^{min}\right)D - P_J D \quad (3.33)$$

3.3.7.2 *The Indifference Point*

In order to determine the demand level at which the total annual costs under both EOQ and JIT are equal, *Z* that stands for the cost difference between JIT and EOQ, in equation (3.33), should be equated with zero. Solving for D (where $D=D_{ind}$), will result in the desired level of demand, therefore,

$$D_{ind} = \frac{HQ_{max}}{2\left(P_J - P_E - \frac{(K + C_S E(s))}{Q_{max}}\right)} \quad (3.34)$$

3.3.7.3 *Maximum JIT purchase price*

Maximum JIT price (P_{Jmax}) is the highest price at which the buyer can use JIT system and still be less costly than the EOQ system. Beyond this price level, EOQ system will be preferred over JIT.

P_{Jmax} can be obtained by setting $Z=0$ in equation (3.33), and solving for P_J where $P_J = P_{Jmax}$, then,

$$P_{J\,max} = \frac{K + C_S E(s)}{Q_{max}} + \frac{HQ_{max}}{2D} + P_E \qquad (3.35)$$

Therefore, for a given demand D, P_{Jmax} is the highest price that the manufacturer can pay to purchase the item on a JIT basis and still have a positive indicator over EOQ purchasing.

The manufacturer utilizing JIT system still can negotiate with the supplier to get the price at which the total annual cost using JIT will be less than using the EOQ. For prices higher than P_{Jmax}, Z will be negative, making EOQ a lower cost alternative.

3.4 Model III

In the first model, a fixed inventory purchase price is studied. In the second model a linear discount purchase price function is studied and a mathematical model comparing the total annual costs under JIT and EOQ is developed in both cases, for the fixed price function as well as for the linear discount purchase price function.

Another discount scheme that will be studied here in this model is the step function price discount scheme, where there are several levels of purchase prices and each lower price level corresponds to a higher quantity range. Whereas; for the same price level, the price is fixed.

The discount is given only for larger quantities. Therefore it is considered as an incentive to the customer to buy larger lot sizes and get the benefit of lower prices.

3.4.1 *Price Functions for EOQ*

The properties of this step function price discount scheme are graphically presented in Figure (3-3).

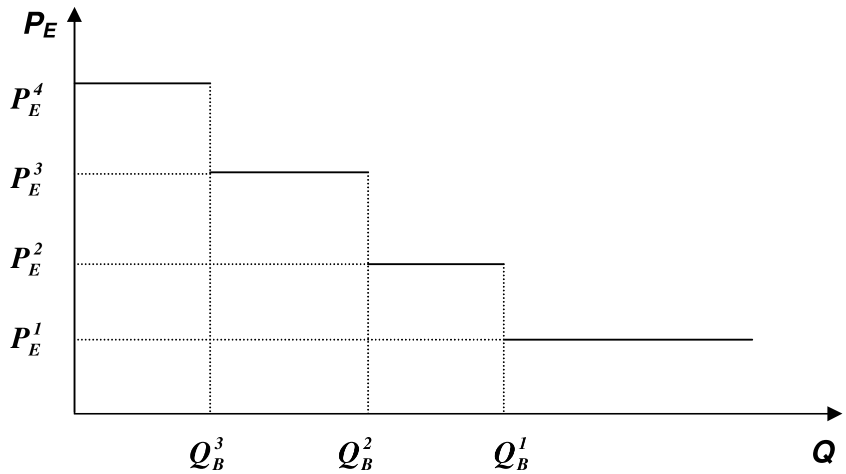

Figure (3-3) EOQ step discount function

Note that the unit price differs for every quantity range, whereas it is fixed for the same range of quantities. The minimum inventory item purchase price level is P_E^1, and its corresponding quantity is the minimum quantity that is needed to get the lowest price level (Q_B^1). Beyond this quantity the price is fixed no matter how large the order quantity is.

Q_B^i is a price break quantity, i.e. the minimum quantity needed to get a discount, and the superscript number corresponds to the price level. The lowest price level in this discount scheme is denoted in this model to be level number 1.

3.4.2 *EOQ Costs*

The same total annual cost, using the EOQ system that are used in the above two models, will be used here in the third model. Therefore, the total annual cost using the EOQ basis consists of: inventory holding cost, inventory ordering cost, stock-out cost, and the delivered goods cost, then,

$$TAC_E = \frac{D}{Q_i}(K) + \frac{Q_i}{2}(H) + \frac{D}{Q_i}[C_s E(s)] + (P_E^i)D \qquad (3.36)$$

Where (i) is the price level, Q_i is the quantity that lies within the range corresponding to the price level P_E^i.

3.4.3 *Determining the EOQ*

The EOQ, that minimizes the sum of the total annual costs for every price level can be determined from the equation below:

$$Q^* = \sqrt{\frac{2D[K + C_s E(s)]}{H}} \qquad (3.37)$$

Note that the unit holding cost *H* is usually expressed as a percentage of the unit price P_E. The more valuable the item held in inventory, the higher the holding cost. Thus the lower the unit price P_E is, the lower *H* is. Conversely, the higher P_E is, the higher *H* is (Krajewski and Ritzman, 2002).

Consequently, every price level should contain a different EOQ. Unfortunately this is not the case when it comes to the determination of the EOQ for each price level, because the resultant EOQ might fall within a different price level other than the correct one making this EOQ a non-feasible quantity.

Since item cost is not constant nor a continuous function of the order quantity, and in order to determine the optimal order quantity, i.e. the quantity that minimizes the sum of the total annual costs, and since the quantity discount model can not be solved directly from calculus, a computational procedure that involves several steps will lead to the optimum order quantity where the minimum cost is proposed.

The following steps are proposed to determine the optimum order quantity:

1. Calculate the economic order quantity using minimum unit cost P_E^1, from equation (3.37).
2. If this quantity is feasible, i.e. it falls within the range for which the cost is correct, it's a valid economic order quantity and will result in the minimum cost for the particular item according to EOQ model. Therefore, it will be taken as the optimal order quantity.
3. If the EOQ calculated in step 1 is not feasible, find the EOQ for every consecutive upper price level until you reach a feasible EOQ.
4. Calculate the total annual cost for the first feasible EOQ and for larger price break quantity at each lower price level (from equation (3.36)), where the price break is the minimum quantity needed to get a discount.
5. The minimum-cost order quantity is the one associated with the lowest cost in step 4, and will be taken as the optimal order quantity.

After determining the optimal order quantity, which model will be the least costly to use, the EOQ system or the JIT system? A mathematical model is developed here to answer the above question. Taking into consideration the two possible cases that may occur here, depending on which quantity is the optimal quantity; the EOQ or the price break quantity.

3.4.4 Case I

If the optimal quantity is the EOQ, then we will consider the price to be fixed, and equals the price corresponding to the range of quantities where the EOQ lies in. Therefore, the price that will be used here will be P_E^{EOQ}, where the superscript EOQ stands for the price level where the EOQ falls in (the optimal quantity).

3.4.4.1 JIT Costs

The annual cost to the buyer under JIT purchasing is the product of the annual demand (*D*) and the unit price (P_J), which is assumed to be fixed, where P_J includes the portion of holding and ordering costs that are passed on to the buyer.

$$TAC_J = P_J D \tag{3.38}$$

3.4.4.2 Cost Difference

Let *Z* represent the difference between the total annual costs of EOQ and JIT models, then:

$$Z = TAC_E - TAC_J \tag{3.39}$$

Substituting $P_E = P_E^{EOQ}$ in equation (3.36) for the total annual cost under EOQ, yielding:

$$TAC_E = \frac{D}{Q}(k) + \frac{Q}{2}(H) + \frac{D}{Q}[C_s E(s)] + \left(P_E^{EOQ}\right)D \tag{3.40}$$

Substituting the EOQ formula from equation (3.37) in equation (3.40), yielding:

$$TAC_E = \sqrt{2DH[K + C_S E(s)}) + P_E^{EOQ} D \tag{3.41}$$

Substituting the above equation for the EOQ total annual cost, and equation (3.38) for JIT total annual cost in equation (3.39) resulting:

$$Z = \sqrt{2DH[K + C_S E(s)} - D\left(P_J - P_E^{EOQ}\right) \qquad (3.42)$$

Now, Let C be the dollar value of the annual demand:

$$C = D\,P_E^{EOQ} \qquad (3.43)$$

Rearranging equation (3.42) by multiplying and dividing the equation by P_E^{EOQ} leads to:

$$Z = \sqrt{\frac{2H[K + C_S E(s)]}{P_E^{EOQ}} C} - \left(\frac{P_J}{P_E^{EOQ}} - 1\right)C \qquad (3.44)$$

3.4.4.3 *The Indifference Point*

For computed values of *Z* that are positive, the JIT system is less costly, whereas, for negative values of *Z* the EOQ system is the least costly system. Setting *Z=0* in equation (3.44) and solving for *C*, where $C = (C_{ind})$, then,

$$C_{ind} = \frac{\dfrac{2H[K + C_S E(s)]}{P_E^{EOQ}}}{\left(\dfrac{P_J}{P_E^{EOQ}} - 1\right)^2} \qquad (3.45)$$

Where, the indifference point C_{ind} is the level of annual demand (in $) at which the total cost of both EOQ and JIT is equal.

Substituting $C_{ind} = D_{ind}\ P_E^{EOQ}$ in equation (3.45) and solving for D_{ind}, where D_{ind} is the annual demand at *Z=0*, resulting:

$$D_{ind} = \frac{2H[K + C_S E(s)]}{\left(P_J - P_E^{EOQ}\right)^2} \qquad (3.46)$$

Note that if the annual demand $D > D_{ind}$ then $Z < 0$, so EOQ model will be more cost effective than JIT model, on the other hand, if $D < D_{ind}$ obviously $Z > 0$, consequently, JIT model will be preferred over EOQ model.

3.4.4.4 Maximum JIT Purchase Price

Maximum JIT price (P_{Jmax}) is the highest price at which the buyer can use the JIT model and still be less costly than the EOQ model, beyond this price, the EOQ model will be preferred over JIT.

P_{Jmax} can be obtained by setting $Z=0$ in equation (3.42), and finding P_J where $P_J=P_{Jmax}$:

$$P_{J\,max} = \sqrt{\frac{2H[\,K + C_S E(\,s\,)]}{D}} + P_E^{EOQ} \qquad (3.47)$$

Prices higher than P_{Jmax}, Z will be negative, making EOQ a lower cost alternative. This gives the JIT buyer a chance to negotiate with the supplier in order to get the best price, i.e. the price less that P_{Jmax}.

3.4.4.5 Maximum Cost Advantage

The cost difference between EOQ and JIT is maximized for a demand level (D_{max}) at which, $dZ / dD=0$, solving this equation for D_{max}, yields the following:

$$D_{max}=D_{ind} / 4 \qquad (3.48)$$

3.4.5 Case II

The second case occurs when the optimal quantity is the price break quantity (O_B), thus its corresponding price level is P_E^{QB}. Therefore, in order to decide whether to switch to the JIT system or not, we have to find out which system will be the least costly, and through following mathematical analysis, we will be able to decide.

3.4.5.1 EOQ Costs

The total annual cost using the EOQ basis consists of: inventory holding cost, inventory ordering cost, stock-out cost, and the delivered goods cost. Therefore,

$$TAC_E = \frac{D}{Q_B}(K) + \frac{Q_B}{2}(H) + \frac{D}{Q_B}[C_s E(s)] + \left(P_E^{QB}\right)D \qquad (3.49)$$

Where subscript (B) stands for the break price quantity.

3.4.5.2 JIT Costs

The annual cost to the buyer under JIT purchasing is the product of the annual demand (D) and the unit price P_J, therefore,

$$TAC_J = P_J D \qquad (3.50)$$

3.4.5.3 Cost Difference

Let Z represent the difference between the costs of EOQ and JIT systems, then:

$$Z = TAC_E - TAC_J \qquad (3.51)$$

Substituting equations (3.49) and (3.50) into equation (3.51) results in:

$$Z = \frac{D}{Q_B}(K) + \frac{Q_B}{2}(H) + \frac{D}{Q_B}[C_s E(s)] + \left(P_E^{QB}\right)D - P_J D \qquad (3.52)$$

3.4.5.4 The Indifference Point

In order to determine the demand level at which the total annual costs under both EOQ and JIT are equal, Z that stands for the cost difference between JIT and EOQ, in equation (3.52) should be equated with zero, and then solving for D (where $D=D_{ind}$), yielding:

$$D_{ind} = \frac{HQ_B}{2\left(P_J - P_E^{QB} - \frac{(K + C_S E(s))}{Q_B}\right)} \tag{3.53}$$

3.4.5.5 Maximum JIT Purchase Price

Maximum JIT price (P_{Jmax}) is the highest price at which the buyer can use the JIT model and still be less costly than the EOQ system, beyond this price the EOQ model will be preferred over JIT.

P_{Jmax} can be obtained by setting $Z=0$ in equation (3.52), and solving for P_J where $P_J=P_{Jmax}$:

$$P_{J\,max} = \frac{K + C_S E(s)}{Q_B} + \frac{HQ_B}{2D} + P_E^{QB} \tag{3.54}$$

So, with a given demand, P_{Jmax} is the highest price that the manufacturer can pay to purchase the item on a JIT basis and still have a positive financial base over EOQ purchasing. Therefore, the buyer utilizing JIT system can negotiate with the supplier to get the price at which the total annual cost using JIT will be less than using the EOQ system, i.e. the price should be less that P_{Jmax}. Whereas, for prices higher than P_{Jmax}, Z will be negative, making EOQ a lower cost alternative.

3.5 Concluding Remarks

In this Chapter three mathematical models are developed taking into consideration three different real life cases. In the first model, the manufacturer deals with suppliers under EOQ that sells on a fixed price basis.

Inventory costs of purchasing under the EOQ system is determined and compared to the costs under the JIT system. The second and the third models are developed on a quantity discount basis. In model two, a linear discount scheme is studied and a mathematical model is developed comparing JIT and EOQ total annual costs. Finally, in the third model a step function discount scheme is discussed and also a mathematical model is developed to compare inventory costs of purchasing under JIT and EOQ system.

In order to illustrate and validate the above models, an example for each model will be presented in the next Chapter. Results will be plotted and discussing those results will conclude the Chapter.

Chapter Four

Comparison Between JIT Purchasing and EOQ Purchasing

(Examples & Discussion)

4.1 Introduction

The three models developed in the previous Chapter will be validated in this Chapter. One of the ways to validate a model is through an example. Therefore, each model will be validated and thus illustrated through an example. A graphical representation of each example's results will be presented. Finally, this Chapter will be concluded with a discussion of the results and concluding remarks.

4.2 Example I

Model I, the model dealing with items having fixed costs or the non-discount model, will be illustrated through this example. Assume that purchasing an item under EOQ model with (P_E)=*\$50/unit*. The estimated annual holding cost per unit (H)=*\$15* (30 percent of the purchase price), and ordering cost (K)=*\$90/order*. If the firm purchases the item on a JIT basis the cost will be (P_J)=*\$52/unit.* Also assume that the expected number of units short per replenishment order cycle ($E(s)$)=*8 units*, and the estimated shortage cost per unit (C_S)=*\$6/unit*.

Therefore, from equation (3.15) in Chapter Three, (D_{ind})=*1035 units*, and the cost advantage of JIT system is maximized when the annual demand equals (D_{max})=*259 units*. Figure (4-1) is a graphical representation of the cost difference between EOQ and JIT (Z) as a function of annual demand (*D*).

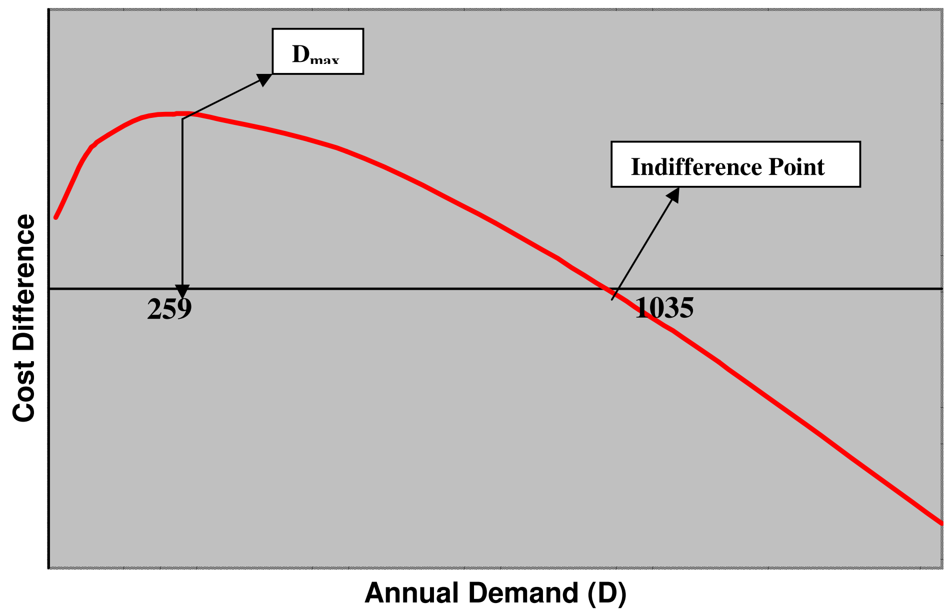

Figure (4-1) Cost difference between EOQ and JIT (for the fixed price model)

Figure (4-1) indicates that as the annual demand increases, the cost difference between EOQ and JIT increases until it reaches a point where the cost difference will be at its maximum value, this point of demand is (D_{max})=*259 units* in this example. Afterwards, the cost difference starts decreasing until it reaches the zero value.

The decrease in the cost difference means that the cost advantage of the JIT system over the EOQ decreases, until the cost difference intersects the demand axis. At this point the cost difference will be zero, i.e. the total annual cost for both EOQ and JIT is the same. This point is called the indifference point, and the demand level at that point is D_{ind}. In this example the value is (D_{ind})=*1035 units.*

Beyond the indifference point, EOQ will be less costly, and will be preferred over the JIT system. The larger the demand beyond D_{ind} is, the more the cost advantage of the EOQ system over JIT will be.

Furthermore, for a given demand such as 900 units/year JIT will be preferred over EOQ as long as P_J is less than (P_{Jmax})=*$52.14*. This is reasonable because P_{Jmax} is the item purchase cost under JIT at the indifference point (where the total annual costs for both JIT and EOQ are the same), beyond this price EOQ will be more cost effective than JIT system.

Now let us study the effect of EOQ costs, i.e. holding, ordering, and stock-out cost on the indifference point. Assume that due to certain economic conditions the holding cost has increased from (H)=*$15* to become ($H$)=$30. Then determining the indifference point results in (D_{ind}) = 2070 units. Note that an increase in the holding cost has widened the range of annual demand favoring the JIT system, and the same goes for ordering cost and stock-out cost.

Since holding, ordering, and stock-out costs are directly proportional to D_{ind}, any increase in any value of these costs will shift the indifference point to the right in Figure (4-1), i.e. the range of annual demand at which JIT will be preferred over EOQ will increase. This is consistent with the expectations that the higher the holding, ordering, and stock-out costs, the more costly the EOQ system will be, and the more JIT system will be preferred over EOQ over a wider range of annual demands.

4.3 Example II

The second case in model two will be illustrated in this example. Case II occurs when the EOQ corresponding to the minimum unit price level (*Q***) is infeasible, whereas, the EOQ calculated over the discount range of order quantities (*Q**) is feasible, and also the total annual cost for *Q** is less than the total annual cost of Q_{max} (the minimum quantity having the lowest unit purchase price).

Let us assume that a manufacturer is considering a choice between EOQ and JIT for purchasing a certain inventory item with an annual demand of 10,000 units. The cost of purchasing the item under JIT system (P_J)=*$50.50/unit*. If this manufacturer purchases the item according to the EOQ model, the pricing strategy offered by the supplier will be as follows:

The delivery price starts at $50/unit. For every additional unit ordered, price will decrease by $0.0004/unit for the entire order lot. The discount is valid for order quantities up to 2,500 ($Q_{max}=2,500$), when the price per unit becomes $49. Beyond this level, the price remains the same. The estimated annual holding cost per unit is $15 (about 30% of the purchase price), and ordering cost is $60 per order. Also assume that the expected number of units short per replenishment order cycle ($E(s)$)=*0.6* units, and the estimated shortage cost per unit (C_S)=*$4/unit*.

That is, (D)=*10,000 units/year*, (P_J)=*$50.50/unit*, ($P_E^o$)=*$50/unit*, (π_E)=*0.0004*, (Q_{max})=*2,500 units*, (P_E^{min})=*$49/unit*, ($H$)=*$15/unit/year*, and (K)=*$60/order*, ($E(s)$)=*0.6 units,* ($C_S$)=*$4/unit.*

In order to assess which system is less costly than the other, we should determine the indifference point value. Then we should compare the demand level at this point with the company's annual demand, if the annual demand is greater than the indifference point, the EOQ system will be more cost effective than JIT, and vice versa.

By applying equation (3.29), the indifference point in this example will be (D_{ind})=5351 units and the cost difference (between EOQ and JIT system) can be maximized when the demand level equals (D_{max})=1450 units. Therefore, for a company having annual demand beyond the indifference point, EOQ system will be a less costly option. In this example it will be better for the company to use the EOQ system rather than JIT, because annual demand, which is (D)=10,000 units, is beyond the indifference point.

Now let's study the effect of increasing one of the EOQ costs. Lets choose the ordering cost for instance. Assume that the conditions for the company remains the same but the ordering cost has increased to become (K)=*$180/order*. Now recalculating the indifference point we find that (D_{ind})= 10,099 units. Note that the range of demand making the JIT system a less costly alternative has increased substantially, and here in our example for the company having an annual demand equals to 10,000 units, JIT system is a more feasible alternative, because the increase in the ordering cost has increased the range of demand levels favoring JIT system over the EOQ system. The indifference point corresponding to (K)=*$60/order* was ($D_{ind}$)=5351 units, while the indifference point corresponding to (K)=*$180/order* is ($D_{ind}$)= 10,099 units, note the effect of increasing the ordering cost on the indifference point.

A clear picture for the first case where (*K*)=*$60/order* can be taken through Figure (4-2).

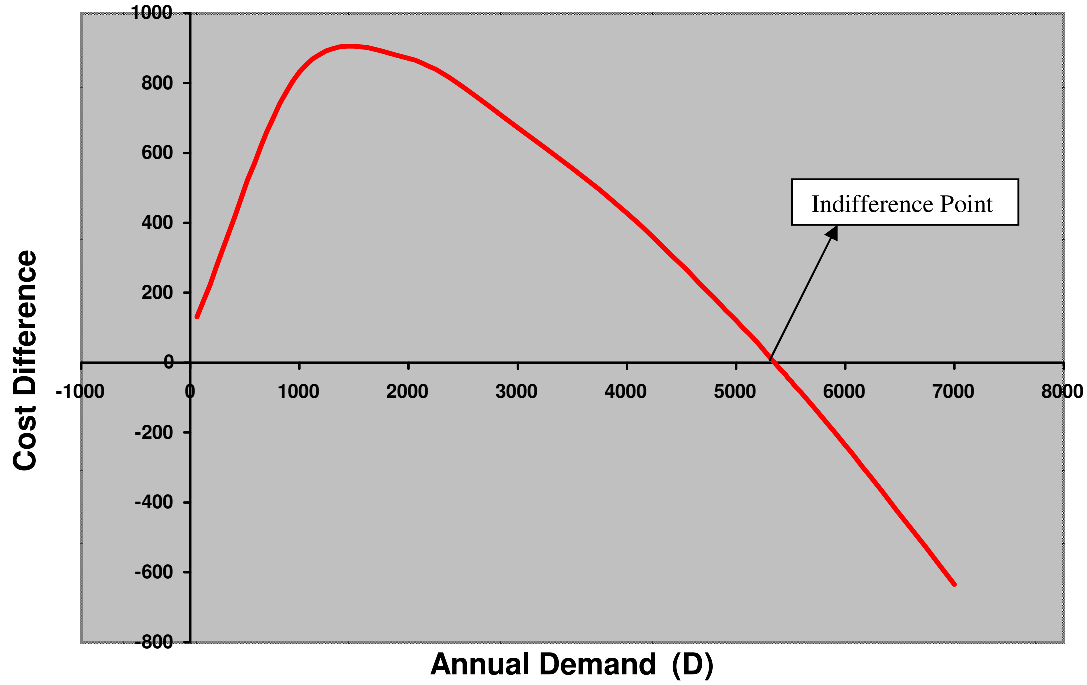

Figure (4-2) Cost difference between EOQ and JIT (for the linear price-discount model)

Figure (4-2) clearly shows the cost difference between EOQ and JIT as a function of annual demand. It indicates that for low levels of demand JIT is more economical. As the demand increases, the cost difference widens to a point where this difference is maximized (D_{max}). For this set of parameters this quantity is 1450 units. As the annual demand increases beyond this point, the cost advantage of JIT begins to fade, until the two costs become equal at a demand level of 5351units, which is the indifference point. For annual demands above this level, EOQ is the more cost effective alternative.

4.4 *Example III*

One of the best ways to illustrate model three is through the following example. Let us assume that a manufacturer is studying whether he should switch to JIT system or not. The EOQ supplier offers a step function discount as follows:

If the order quantity is less than 300units, the unit price will be $60.00, quantities from 300 up to 499 the unit price will be $58.80. If the quantity is 500 or more the unit price will be $57.00.

The annual demand estimate for this item is 936 units; its ordering cost is $45 per order. Holding cost is 25% of the item's unit price. Also assume that the expected number of units short per replenishment order cycle ($E(s)$)$=0.6$ units, and the estimated shortage cost per unit (C_S)$=\$4/unit$. If he purchases the item on a JIT basis the cost will be (P_J)=$60.6.

Following the computational procedure suggested in this model (model III), then:

Calculating the EOQ corresponding for the minimum unit cost ($57.00) results in a non-feasible quantity ($EOQ_{\$57}$ =77 units which is less than 500 units, so it does not fall within the range corresponding to $57 unit price). The next step is to calculate the first feasible EOQ starting with the second lowest price level, which is (P_E)= $58.8. The economic order quantity corresponding to this level of price is also non-feasible since ($EOQ_{\$58.8}$)=78 units does not fall within the range where the price is correct. Now after that we calculate the EOQ corresponding to the upper price level we find that the EOQ in this level is feasible, therefore, ($EOQ_{\$60.0}$)=77 is a feasible quantity.

So the first feasible EOQ is the one whose price level is $60 per item. Now, we have to calculate the total annual cost for this feasible EOQ and also for the price break quantities for lower price level, which results in: (TAC_{EOQ})= $57,314, (TAC_{300})= $57,390, (TAC_{500})= $57,003.

Since the optimal order quantity is the one that is associated with the minimum cost, the price break quantity 500 will be taken as the optimal order quantity (OOQ) here, because it has the least annual cost among the others.

In order to examine whether the manufacturer should switch to JIT system or not, we will find D_{ind} and compare it with the company's annual demand. The indifference point in this example using equation (3.53) equals (D_{ind})= 1016 units. The company's annual demand is less than D_{ind}, consequently, the less costly alternative under such company's circumstances will be the JIT system, and therefore switching to JIT system will reduce company's costs.

Calculating the highest price that the manufacturer can pay and still be better of using JIT over the EOQ resulted in (P_{Jmax})= $60.9, note that P_{Jmax} is greater than P_J in this example. Therefore, the company's less costly option will be JIT, and this result is consistent with the case when we compared D_{ind} with the annual demand.

Figure (4-3) is a graphical representation of the cost difference between EOQ and JIT (Z) as a function of annual demand (D). As we can visualize from the Figure the relationship between the cost difference and the annual demand is a linear function. This linear function came from the fact that the price break quantity is a fixed quantity (constant) that has no formula to be calculated through, so substituting the price break in the cost difference

(Z) formula, i.e. equation (3.52), will result in a linear function. As shown in the above Figure, the amount of demand that makes the cost difference equals to zero (the point at which the total annual costs under JIT and EOQ is the same) is 1016units. This point is the indifference point and the demand at this point is D_{ind}.

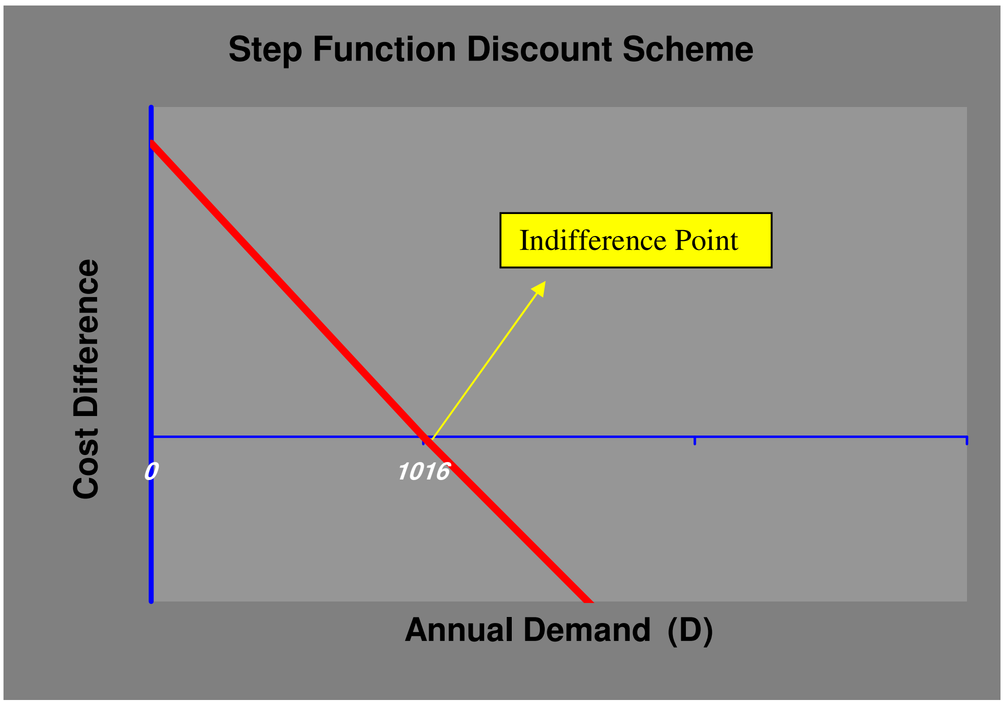

Figure (4-3) Cost difference between EOQ and JIT (for the step function price-discount model)

4.5 *Discussion of The Results*

The above examples results will be discussed in the following Sections.

4.5.1 *Fixed Delivery Price Model*

Ideally, traditional costs associated with the EOQ model are either eliminated or substantially reduced under the JIT philosophy, therefore, if a just in time supplier delivers the goods with an item price less than the EOQ delivery price, i.e. $P_J < P_E$, then according to equation (3.11), the cost difference will be positive, making the total annual cost under EOQ system more costly than JIT system. Under such ideal circumstances, JIT will be the less costly alternative.

Unfortunately, this is not the case when it comes to real life, since some of the JIT supplier costs will be passed on to the buyer. An important finding by Romero (1991) was that the extent of JIT purchasing has a direct effect on reducing costs in buyer organizations but not in supplier organizations. Previous research found that JIT purchasing could result in inventory costs being transferred from buyers to sellers. Therefore, its reasonable to assume that, in the absence of a holistic system in which both the supplier and manufacturer operate under a perfect JIT system, the supplier would pass some of the costs to the JIT manufacturer in the form of higher prices to at least partially reflect ordering and holding costs that have been eliminated. This translates into higher per unit purchase price for the JIT manufacturer (Willis and Huston, 1990).

The indifference point is the break-even demand at which both total annual costs under JIT and EOQ are the same. Therefore, the only case where the JIT will be superior and less costly is when the annual demand is less than D_{ind} (the indifference point), making the cost difference (Z) positive. As annual demand increases past the indifference point, EOQ becomes the preferred method for controlling inventory orders. The cost difference between JIT and EOQ is maximized for a demand level D_{max} (equation (3.17)), at which

JIT has the highest cost advantage over EOQ. P_{Jmax} is the upper limit purchase price above which JIT will be more costly than EOQ.

Now the effect of every cost that constitute the total annual cost on the indifference point will be discussed:

- **Impacts of holding:**

  As can be seen from the D_{ind} formula in equation (3.15), the break-even demand, the demand at which both the EOQ and JIT costs are the same, is directly proportional to the holding cost. The higher the holding cost, the wider the range of demand at which JIT is less costly than EOQ and more preferred.

- **Impacts of ordering cost:**

  Equation (3.15) shows that items with high ordering cost (K) have a wider range of demand over which JIT is preferable. This is consistent with the expectations that under JIT efforts should be made to lower the ordering cost. Having the supplier located near the JIT manufacturer, transportation costs should be much less resulting in lower ordering cost.

- **Impacts of stocking-out cost:**

  As can be seen from equation (3.15) regarding the indifference point, the break-even demand is directly proportional to the stock-out cost. Any increase in the stock-out cost will cause an increase in the break-even demand level resulting in widening the range of annual demands favoring the JIT system.

- **Impacts of purchase price:**

  As can be seen from equation (3.15) the larger the difference between JIT and EOQ purchase prices, the smaller the range of annual demand for which JIT is preferred.

Equation (3.16) establishes the threshold price level below which the total annual cost under JIT will be lower than EOQ for a given level of demand and other operating conditions. JIT is less costly than EOQ only as long as the JIT purchase price is below P_{Jmax}. Equation (3.16) also shows that as the level of demand increases, P_{Jmax} will decrease.

4.5.2 *Linear Price-Discount Model*

Three cases are likely to occur in Model II. Case one is when the optimal quantity is the EOQ corresponding to the minimum purchase price level, i.e. the price corresponding to the range of quantities above Q_{max}. This case is consistent with the first model, considering unit purchase price is equal to $P_E = P_E^{\min}$. Therefore the same comments on the fixed model can be used here in this case.

The second case occurs when the total annual cost corresponding to Q^* is less than the total annual cost of Q_{max}. As can be seen from the formula used to calculate the break-even demand in equation (3.30), the holding cost represented by (H), the ordering cost (K), and the stock-out cost represented by $(C_S E(s))$ are directly proportional to the break-even demand. But note that the term representing both the stock-out as well as the ordering cost $(K + C_S E(s))$ is also in the denominator, but it is multiplied by the discount ratio, which is normally very small (in our example it was 000.4), making the effect on the indifference point much less than it is in the numerator. Therefore, we can consider an increase in the ordering cost or stock-out cost will cause an increase in the indifference point.

Figure (4-2) is a graphical representation of the cost difference between EOQ and JIT (*Z*) as a function of the annual demand. It indicates that for low levels of demand JIT is more economical. As demand increases, the cost difference widens rapidly until a point where this difference is at its maximum (D_{max}). As the annual demand increases beyond this point, the cost advantage of JIT begins to fade, until the two costs become equal. For annual demand above this level, EOQ is the more cost effective.

The third case in this model (model II) occurs when the optimal order quantity (OOQ) is Q_{max}. As can been seen from equation (3.35) the holding, ordering, and stock-out cost are directly proportional to the break-even demand. Therefore, the higher these costs are, the wider the range of annual demand at which JIT will be the less costly alternative.

4.5.3 Step Function Price Discount Model

Model III consists of two cases, the first case where the optimal order quantity is the EOQ. This case is consistent with the first model, at which we take the price level where the EOQ lies as the fixed price level and then we proceed with model I. Therefore, all the notes taken on model I also can be applied in this case regarding the effect of the holding, ordering, and stock-out costs on the indifference point, and thus on the range of annual demands where the JIT system will be preferred over the EOQ system and vice versa.

Case II, in this model is the case when a price-break quantity is taken as the optimal order quantity. This case is consistent with the case when Q_{max}

in model II case III is taken as the optimal order quantity, therefore, the same points discussed there applies here in this case.

For the three models we can note that for relatively low levels of annual demands JIT system is preferred over the EOQ system. Whereas, EOQ system has the cost advantage for an item with higher demand levels.

4.6 Concluding Remarks

In this Chapter the models developed in the previous Chapter has been validated through several examples. The results are discussed and some graphs are used to present and illustrate those results.

In the next Chapter, those three models developed will be computer programmed with user-friendly interfaces. The interfaces of the program along with flow charts expressing the steps taken in developing the program will be presented.

Chapter Five

Comparison Between JIT Purchasing and EOQ Purchasing

(Computer Implementation)

5.1 *Introduction*

One of the best ways to enhance sound management decisions through the development of mathematical models is to establish a user-friendly program that contains the developed models. A user-friendly program has been developed in this Chapter that includes the three models. The interfaces of the program will be presented in this Chapter showing the inputs along with the results that come out of the program.

The software or programming language that is used to establish this program was LabView (Graphical Programming for Instrumentations). The specifications of the computer that this software is installed in and thus the development of this program is through: Pentium I PC, 200 MHz processor, 6 GB Hard drive storage capacity, and with 64 MB RAM.

This Chapter begins with the characteristics of the program. Modules, sub-modules, and flow charts (to illustrate how each module or sub-module works) for each model will be presented. Finally closing remarks will conclude this Chapter.

5.2 *Main Characteristics of the Program*

The main characteristics of the program include:

1. User-friendly interfaces are used.
2. Easy to use, and easy to input the required parameters.
3. Flexible, i.e. required parameters can be fed and also modified easily in the same window.
4. Outputs include almost all the parameters that are discussed in Chapter three.
5. Relationship between cost difference and the annual demand is presented graphically as an output for model I.
6. Investigation of the effect of changing any parameter on any output can be conducted easily using this program.
7. Comparison between total costs for both JIT and EOQ systems is carried out.
8. Deciding under company's circumstances which inventory system (EOQ or JIT) is less costly to use.
9. Determination of the level of annual demand at which the total cost of JIT and EOQ are equal is conducted.
10. The user can enter as many price levels in the step function sub-module as he can; no constraints on the number of price levels are placed.

5.3 *Modules and Sub-Modules*

The program consists of one main module and three other sub-modules. The main module and the sub-modules will be discussed in the coming Sections.

5.3.1 *Main Module*

The program consists of a main module called (Main), the main purpose of this module is to choose what pricing scheme is to be used and consequently what sub-module should be activated. Therefore, the proper relevant interface should be opened to the user in a new window. However, the user is the one who is responsible for making this decision by choosing what pricing policy the supplier offers to the company. The main menu that shows the main module interface is shown in Figure (5-1).

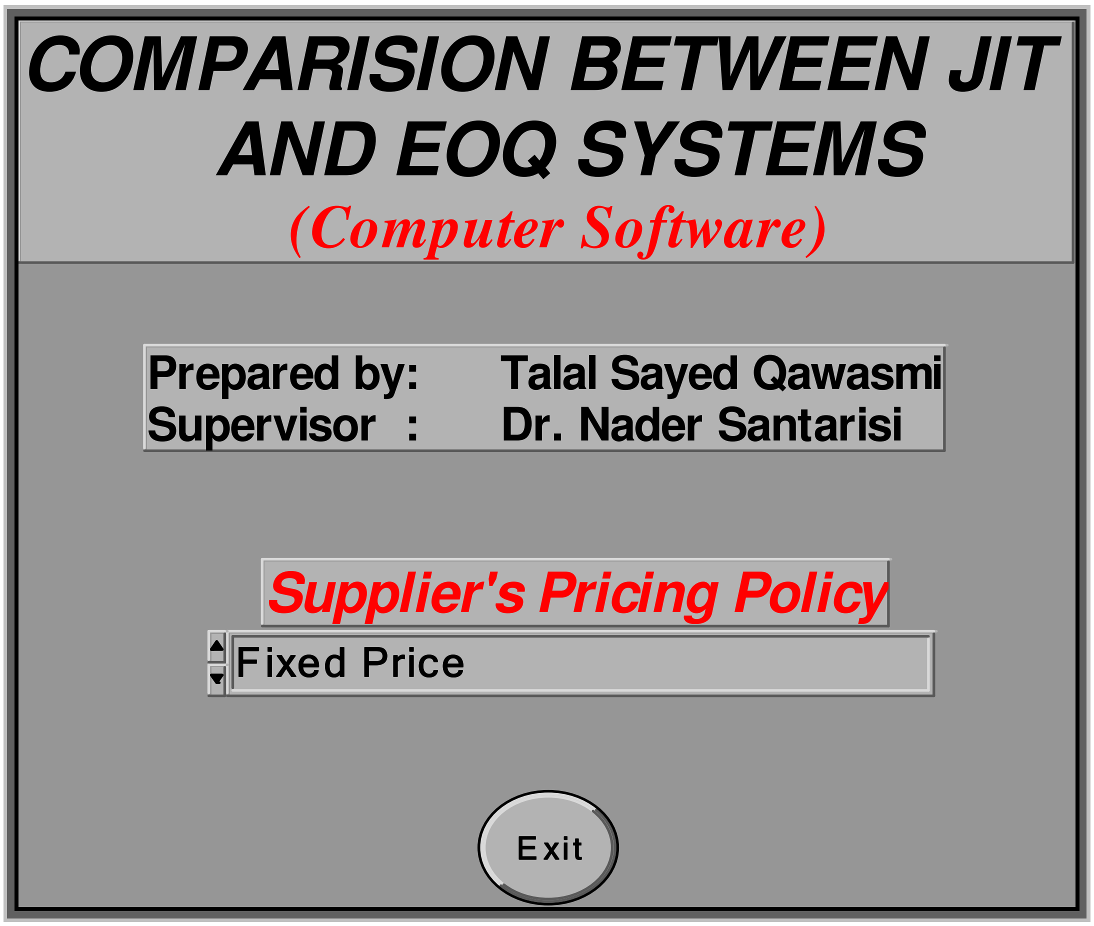

Figure (5-1)Main menu interface

As can be seen in the above Figure the only input that the user is asked to feed the program in the main menu is the supplier's pricing policy. A drop down menu is displayed and the user is asked to choose among three policy alternatives: fixed price policy, linear discount policy, or step function discount policy.

After feeding the program with the pricing policy, the main module activates one of a three sub-modules. A tree diagram showing the main module and sub-modules used in the software is presented in Figure (5-2).

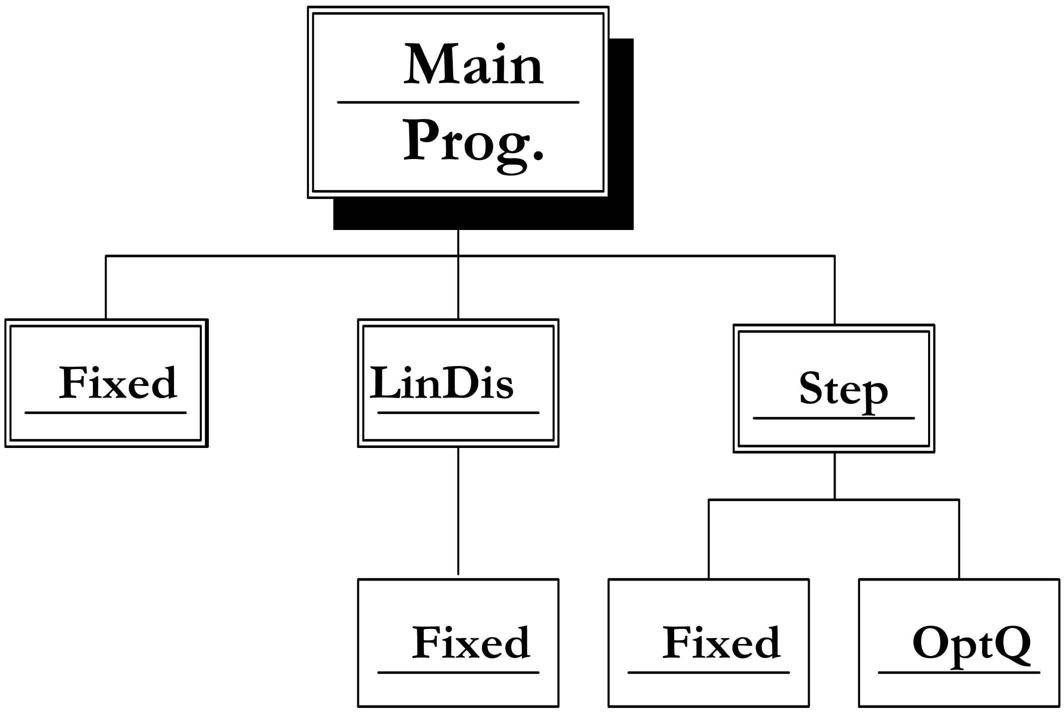

Figure (5-2) Modules and sub-modules

As shown in Figure (5-2), the main module consists of three sub-modules. These modules are: Fixed, LinDis, and Step. In the following Sections these sub-modules will be discussed.

5.3.2 Fixed Sub-Module

Fixed; stands for the fixed price model (model I), i.e. item's delivery price is not subjected to any discount no matter how large the order quantity is. This is the first sub-module established. Figure (5-3) shows a flow chart explaining how this sub-module works and what exactly this sub-module does.

As can be seen in the Figure, the first thing that the Fixed sub-module does is calculating the EOQ. Then according to model I, the formula corresponding to the indifference point was fed into the program and will be used as the second step to calculate the level of demand at the indifference point, i.e. calculating D_{ind}. This value will be compared to the company's annual demand.

If company's annual demand is less than the indifference point, one of the results that will appear on the output screen will be (Switch to JIT system), this means JIT will be the less costly alternative. Whereas, if D_{ind} is less than the company's annual demand the message that will be displayed on the results part of the screen is: (Use EOQ system).

If the less costly alternative system is the EOQ system, the EOQ shall be used as the ordering quantity. Another quantity calculated in this sub-module, in this case (where the less costly alternative is the EOQ system) is the time between orders (*TBO*). Time between orders for a particular lot size is the average elapsed time between receiving (or placing) replenishment orders of Q units (Krajewski and Ritzman, 2002). Expressed as a fraction of a year, the *TBO* is simply EOQ (or the optimal order quantity used in the model) divided by the annual demand. Here in the program the time is expressed in months, therefore the *TBO* here will be the EOQ divided by the annual demand, and the result is multiplied by 12 months/year, therefore,

$$TBO_{EOQ} = (EOQ / D)\ 12\ months/year \qquad (5\text{-}1)$$

Finally the relationship between the cost difference between EOQ and JIT (*Z*) will be graphically plotted, as a function of the annual demand, and the graph will be presented as an output. After completing the run, the results will be displayed on the same input window, as shown in Figure (5-4).

Model I
Fixed Price Model

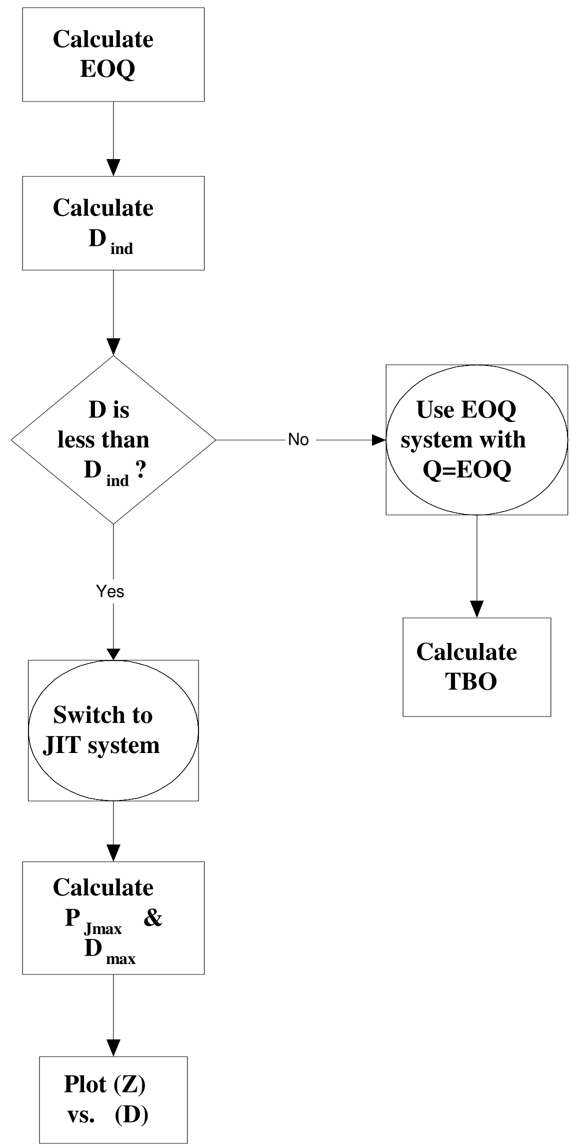

Figure (5-3) Fixed sub-module

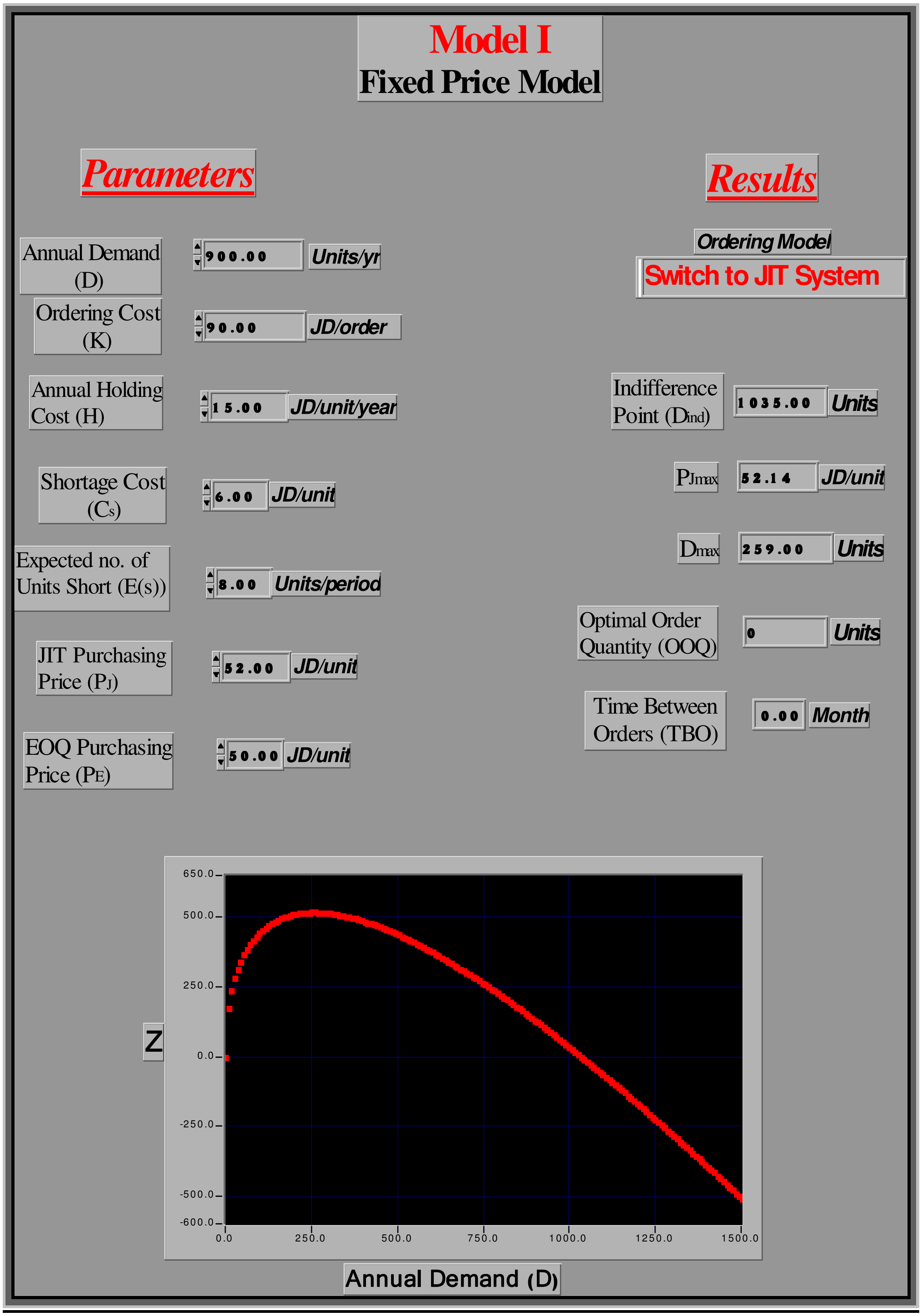

Figure (5-4) Model I program's interface

5.3.3 LinDis Sub-Module

The second sub-module is the LinDis sub-module, which stands for the linear discount model (model II). Starting with the minimum price, this sub-module calculates the EOQ (Q^{**}), then compares the result with Q_{max}, to Figure out whether Q^{**} is a feasible quantity or not. If it is feasible then Fixed sub-module will be activated, and Q^{**} will be fed to the fixed sub-module as the optimal order quantity (OOQ).

If Q^{**} is not a feasible quantity, the program will be asked to calculate Q^{*} (the EOQ that belongs to the discount area) along with the total annual costs (*TAC's*) for both Q^{*} and Q_{max}. The quantity corresponding to the minimum order quantity will be taken as the OOQ (optimal order quantity).

If the optimal order quantity is Q^{*}, the program will calculate the indifference point (D_{ind}), and then compare the results with the company's annual demand. If company's annual demand is less than D_{ind} the result will be (Switch to JIT system), otherwise, the message displayed will be (Use EOQ system), and *TBO* will be calculated and will be shown as one of the results.

If Q_{max} was the quantity with the least *TAC*, D_{ind} will be calculated and then compared to the annual demand. As in the previous case the same procedure will be taken and the only difference here is that the equations fed into the program in order to calculate the D_{ind} are different according to the corresponding equations in each different case in Chapter Three. This procedure is shown in Figure (5-5). The program's interface including the inputs (parameters) and outputs (results) are shown in Figure (5-6).

Model II
Linear Discount Model

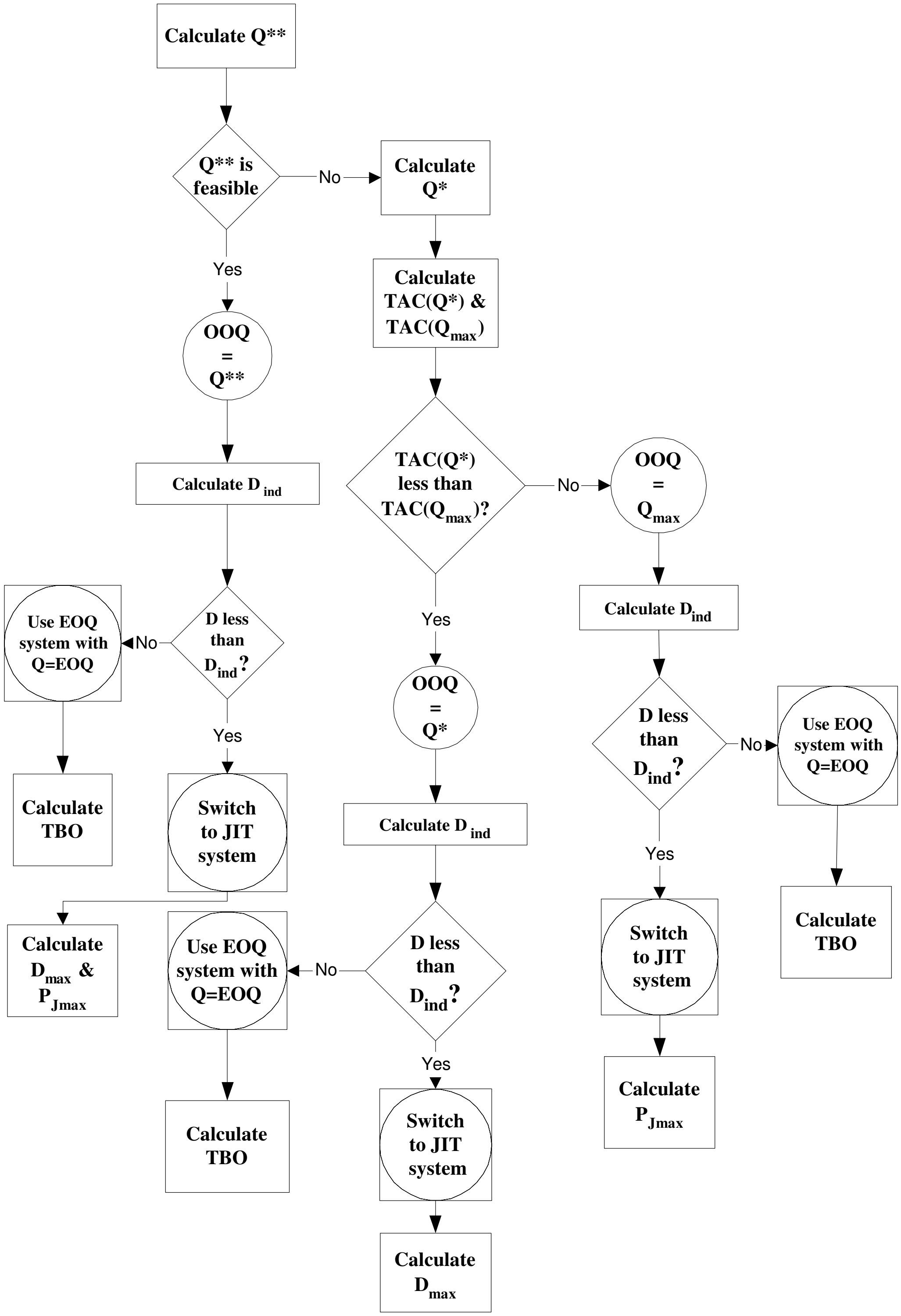

Figure (5-5) LinDis sub-module

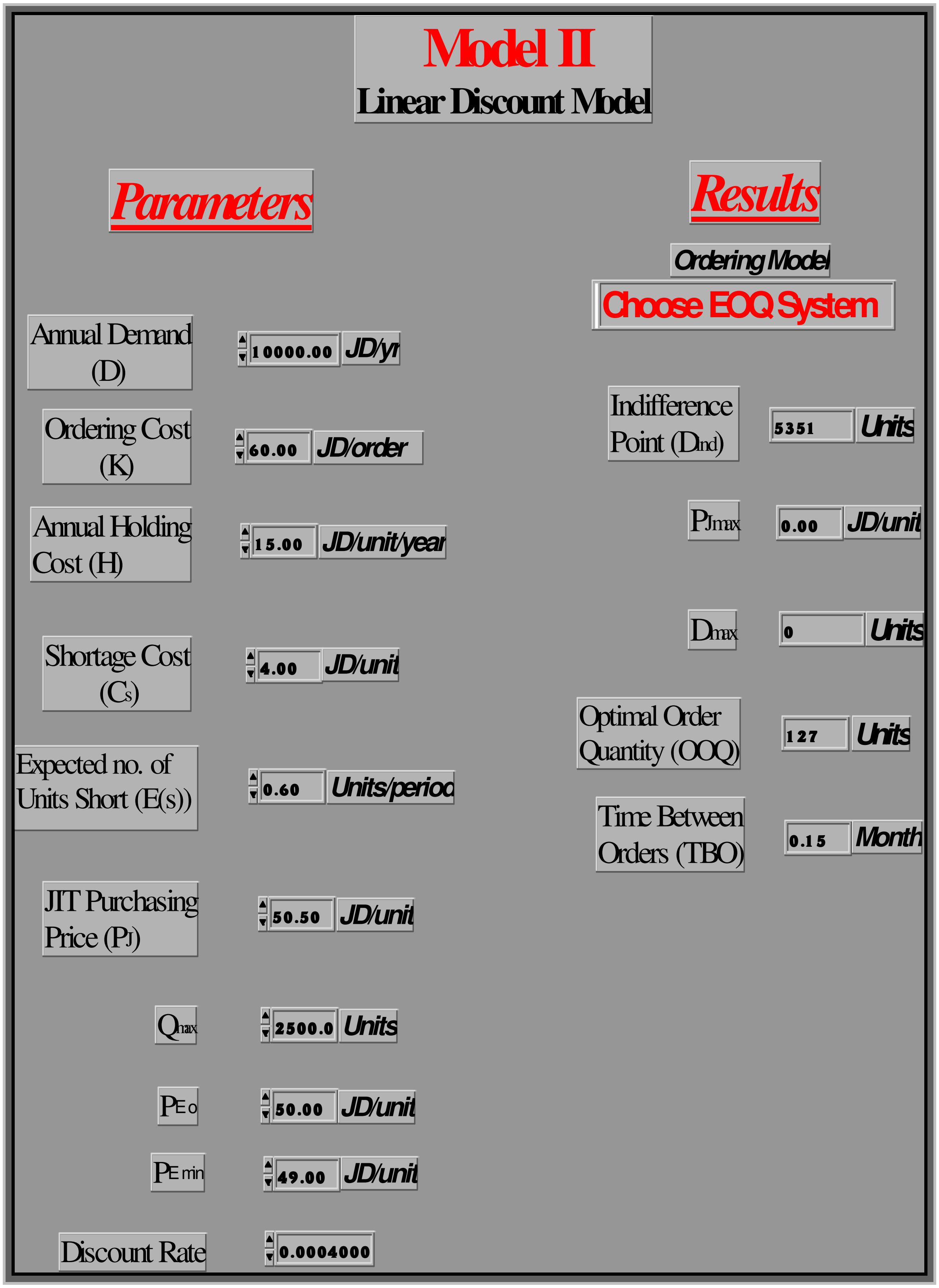

Figure (5-6). Model II program's interface

5.3.4 *Step Sub-Module*

Step refers to the step function discount i.e. Model III. The user is asked to enter the parameters along with the step function pricing parameters in the input screen once he chooses the step function pricing scheme as the supplier's pricing policy in the main menu. This sub-module includes two other sub-modules; the Fixed sub-module, which is the first sub-module mentioned in this context and is discussed in previous Sections, and the *OptQ* sub-module.

The main purpose of the latter sub-module (*OptQ*) is to calculate the economic order quantities starting with the minimum price level until a feasible EOQ is determined, and then determine the total annual costs of the break price quantities (for price levels lower than the first feasible EOQ price level) and also for the first feasible EOQ. The resulting TAC's will be fed to the program through the *OptQ* sub-module.

After having these TAC's on hand, the program compares these total annual costs and the one with the least cost will be considered the OOQ (optimal order quantity) as shown in Figure (5-7). The OOQ here could be the EOQ or the price break quantity (Q_B), in either case, the corresponding indifference point will be calculated and compared to company's annual demand, and the program will show in the results which system will be better to use, the EOQ or the JIT system as shown in Figure (5-8).

Model III
Step Function Model

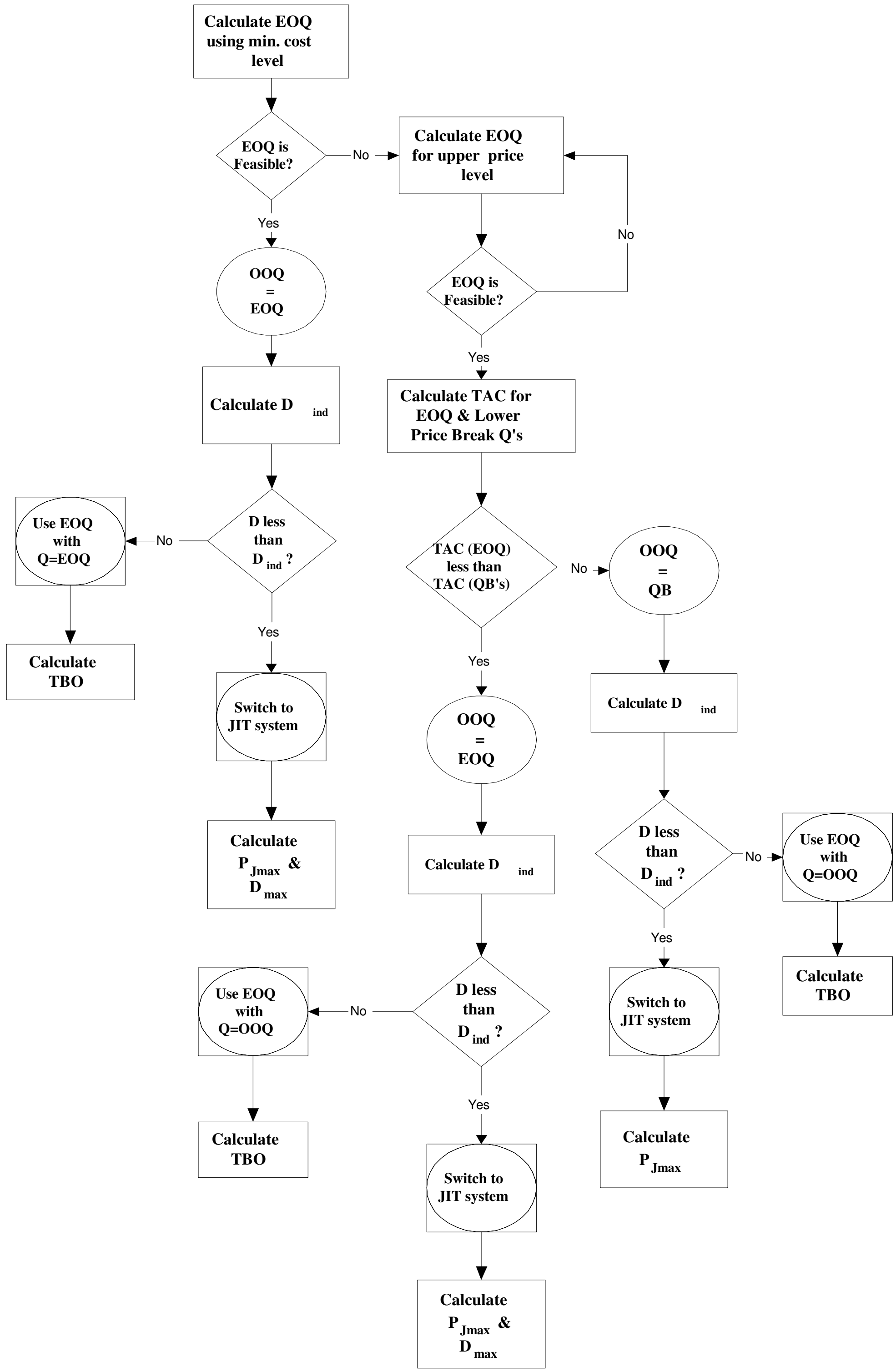

Figure (5-7) Step sub-module

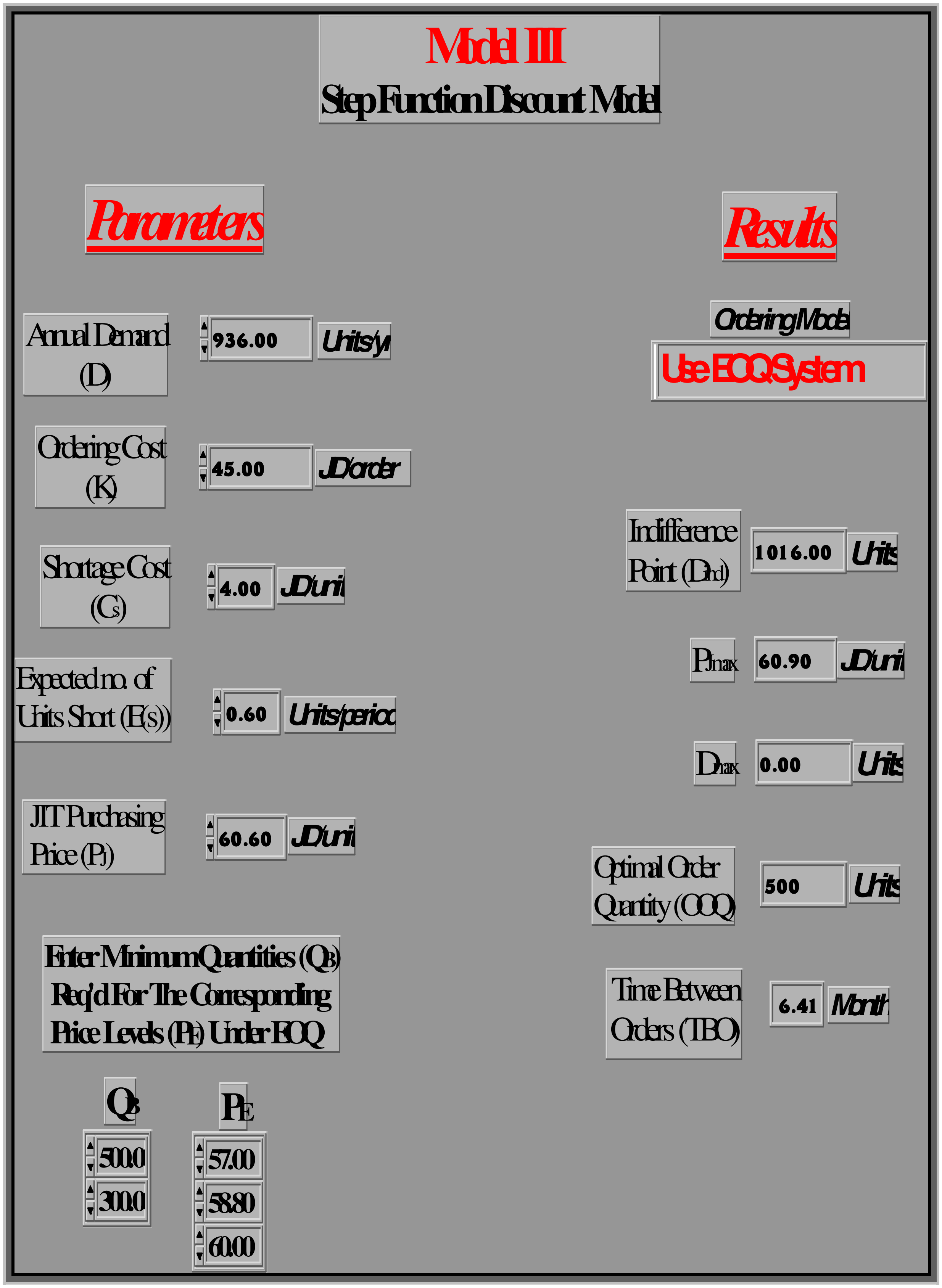

Figure (5-8) Model III program's interface

5.4 *Concluding Remarks*

In this Chapter the developed program is presented along with its interfaces and the characteristics of the software are listed. Afterwards, modules and sub-modules used in the software are discussed. Flow charts are used to express and illustrate each sub-module.

In the next Chapter conclusions drawn will be listed along with the recommendations future researchers.

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ملخـــــــــــــــص

المقارنة بين الشراء بنظام التوقيت الدقيق و نموذج طلب الكميَة الاقتصادية: نمذجة رياضيَة

إعداد

طلال سيد ابراهيم القواسمي

المشرف

الدكتور نادر سعيد السنتريسي

نتيجة للتوجه نحو مجاراة المنافسة العالمية المتزايدة قامت العديد من الشركات الخدمية و الصناعية بزيادة تركيزها على إرضاء المستهلك و نوعية منتجاتها، و لمواجهة هذه التحديات، اضطرت هذه الشركات إلى محاولة إيجاد طرق للتقليل من التكاليف وتحسين نوعية منتجاتها وتلبية متطلبات المستهلكين المتغيرة بشكل مستمر. و لقد كان أحد الحلول الناجحة هو التبني لنظام التوقيت الدقيق، والذي يتضمن العديد من المساحات الوظيفية في الشركة مثل التصنيع و الهندسة و التسويق و الشراء.

إن عدد الشركات التي عملت على إعادة تقييم استراتيجياتها الإنتاجية وسياساتها الشرائية قد ازدادت منذ أوائل الثمانينات، مما أدى تزايد الحاجة إلى نموذج رياضي يكمّم ومن ثمّ يقارن بين التكاليف المتعلقة بنظام التوقيت الدقيق ونظام طلب الكمية الاقتصادية المستخدمة في معظم الشركات و بشكل تقليدي، و يساعد هذا النظام في تحديد مدى قابلية الشركة إلى التحوّل إلى نظام التوقيت الدقيق .

لقد تمّ دراسة تكاليف المخزون الشرائية باستخدام نظام طلب الكمية الاقتصادية، والتي تناولت السعر الثابت و الخصم الخطي والخصم الدرجي كسياسات تسعيرية، فحدّدت وقورنت مع تكاليف نظام التوقيت الدقيق من خلال تطويرثلاث نماذج رياضيّة.

لقد وجدت الدراسة نقطة المساواة (مستوى الطلب الذي تكون فيه التكاليف متساوية) بين النظامين، وقد عرّفت تحت أي الظروف يكون فيها أحد النظامين أفضل من الآخر من جانب التكلفة، و من ثمّ تمّ تأسيس برنامج سهل الاستخدام و يتضمن النماذج المطوّرة.

أوضحت النتائج بانه في حالات مستويات الطلب المنخفضة نظام التوقيت الدقيق هو الطريقة المفضلة، بينما نظام طلب الكمية الاقتصادية يمتلك أفضلية التكلفة للسلعة مع طلب مرتفع. كما وتتنبأ النماذج بأنه كلما ازدادت الأسعار المتعلقة بنموذج طلب الكمية الاقتصادية (تكاليف التخزين، الطلب، ونفاذ الكمية) ازدادت نقطة المساواة، و ازداد أيضا اتساع مدى الطلبات السنوية التي يكون فيها نظام التوقيت الدقيق هو الأفضل من ناحية التكاليف.